Introduction to the performance standards for *Mathematics*

Building directly on the National Council of Teachers of Mathematics (NCTM) Curriculum Standards, the Mathematics performance standards present a balance of conceptual understanding, skills, and problem solving.

The first four standards are the important conceptual areas of mathematics:

- **M1 Arithmetic and Number Concepts;**
- **M2 Geometry and Measurement Concepts;**
- **M3 Function and Algebra Concepts;**
- **M4 Statistics and Probability Concepts.**

These conceptual understanding standards delineate the important mathematical content for students to learn. To demonstrate understanding in these areas, students need to provide evidence that they have used the concepts in a variety of ways that go beyond recall. Specifically, students show progressively deeper understanding as they use a concept in a range of concrete situations and simple problems, then in conjunction with other concepts in complex problems; as they represent the concept in multiple ways (through numbers, graphs, symbols, diagrams, or words, as appropriate) and explain the concept to another person.

This is not a hard and fast progression, but the concepts included in the first four standards have been carefully selected as those for which the student should demonstrate a robust understanding. These standards make explicit that students should be able to demonstrate understanding of a mathematical concept by using it to solve problems, representing it in multiple ways (through numbers, graphs, symbols, diagrams, or words, as appropriate), and explaining it to someone else. All three ways of demonstrating understanding—use, represent, and explain—are required to meet the conceptual understanding standards.

Complementing the conceptual understanding standards, M5 - M8 focus on areas of the mathematics curriculum that need particular attention and a new or renewed emphasis:

- **M5 Problem Solving and Reasoning;**
- **M6 Mathematical Skills and Tools;**
- **M7 Mathematical Communication;**
- **M8 Putting Mathematics to Work.**

Establishing separate standards for these areas is a mechanism for highlighting the importance of these areas, but does not imply that they are independent of conceptual understanding. As the work samples that follow illustrate, good work usually provides evidence of both.

Like conceptual understanding, the definition of problem solving is demanding and explicit. Students use mathematical concepts and skills to solve nonroutine problems that do not lay out specific and detailed steps to follow; and solve problems that make demands on all three aspects of the solution process—formulation, implementation, and conclusion. These are defined in M5, Problem Solving and Reasoning.

The importance of skills has not diminished with the availability of calculators and computers. Rather, the need for mental computation, estimation, and interpretation has increased. The skills in M6, Mathematical Skills and Tools, need to be considered in light of the means of assessment. Some skills are so basic and important that students should be able to demonstrate fluency, accurately and automatically; it is reasonable to assess them in an on-demand setting, such as the New Standards reference examination. There are other skills for which students need only demonstrate familiarity rather than fluency. In using and applying such skills they might refer to notes, books, or other students, or they might need to take time out to reconstruct a method they have seen before. It is reasonable to find evidence of these skills in situations where students have ample time, such as in a New Standards portfolio. As the margin note by the examples that follow the performance descriptions indicates, many of the examples are performances that would be expected when students have ample time and access to tools, feedback from peers and the teacher, and an opportunity for revision. This is true for all of the standards, but especially important to recognize with respect to M6.

M7 includes two aspects of mathematical communication—using the language of mathematics and communicating about mathematics. Both are important. Communicating about mathematics is about ideas and logical explanation. The travelogue approach adopted by many students in the course of describing their problem solving is not what is intended.

M8 is the requirement that students put many concepts and skills to work in a large-scale project or investigation, at least once each year, beginning in the fourth grade. The types of projects are specified; for each, the student identifies, with the teacher, a clear purpose for the project, what will be accomplished, and how the project involves putting mathematics to work; develops a question and a plan; writes a detailed description of how the project was carried out, including mathematical analysis of the results; and produces a report that includes acknowledgment of assistance received from parents, peers, and teachers.

The examples

The purpose of the examples listed under the performance descriptions is to show what students might do or might have done in achieving the standards, but these examples are not intended as the only ways to demonstrate achievement of the standard. They are meant to illustrate good tasks and they begin to answer the question, "How good is good enough?" "Good enough" means being able to solve problems like these.

Each standard contains several parts. The examples below are cross-referenced to show a rough correspondence between the parts of the standard and the examples. These are not precise matches, and students may successfully accomplish the task using concepts and skills different from those the task designer intended, but the cross-references highlight examples for which a single activity or project may allow students to demonstrate accomplishment of several parts of one or more standards.

The purpose of the samples of student work is to help explain what the standards mean and to elaborate the meaning of a "standard-setting performance." Few pieces of work are so all-encompassing as to qualify for the statement, "meets the standard." Rather, each piece of work shows evidence of meeting the requirements of a selected part or parts of a standard. Further, most of these pieces of work provide evidence related to parts of more than one standard. It is essential to look at the commentary to understand just how the work sample helps to illuminate features of the standards.

Resources

We recognize that some of the standards presuppose resources that are not currently available to all students. The New Standards partners have adopted a Social Compact, which says, in part, "Specifically, we pledge to do everything in our power to ensure all students a fair shot at reaching the new performance standards...This means that they will be taught a curriculum that will prepare them for the assessments, that their teachers will have the preparation to enable them to teach it well, and there will be an equitable distribution of the resources the students and their teachers need to succeed."

The NCTM standards make explicit the need for calculators of increasing sophistication from elementary to high school and ready access to computers. Although a recent National Center for Education Statistics survey confirmed that most schools do not have the facilities to make full use of computers and video, the New Standards partners have made a commitment to create the learning environments where students can develop the knowledge and skills that are delineated here. Thus, M6, Mathematical Skills and Tools, assumes that students have access to computational tools at the level spelled out by NCTM. This is not because we think that all schools *are* currently equipped to provide the experiences that would enable students to meet these performance standards, but rather that we think that all schools *should be* equipped to provide these experiences. Indeed, we hope that making these requirements explicit will help those who allocate resources to understand the consequences of their actions in terms of student performance.

The elementary school performance standards are set at a level of performance that is approximately equivalent to the end of fourth grade. It is expected, however, that some students might achieve this level earlier and others later than this grade. The work samples that follow, however, were all done by students who were, in fact, in the fourth grade.

Performance Descriptions *Mathematics*

M Arithmetic and Number Concepts

The student demonstrates understanding of a mathematical concept by using it to solve problems, representing it in multiple ways (through numbers, graphs, symbols, diagrams, or words, as appropriate), and explaining it to someone else. All three ways of demonstrating understanding—use, represent, and explain—are required to meet this standard.

The student produces evidence that demonstrates understanding of arithmetic and number concepts; that is, the student:

M1 a Adds, subtracts, multiplies, and divides whole numbers, with and without calculators; that is:

- adds, i.e., joins things together, increases;
- subtracts, i.e., takes away, compares, finds the difference;
- multiplies, i.e., uses repeated addition, counts by multiples, combines things that come in groups, makes arrays, uses area models, computes simple scales, uses simple rates;
- divides, i.e., puts things into groups, shares equally; calculates simple rates;
- analyzes problem situations and contexts in order to figure out when to add, subtract, multiply, or divide;
- solves arithmetic problems by relating addition, subtraction, multiplication, and division to one another;
- computes answers mentally, e.g., 27 + 45, 30 x 4;
- uses simple concepts of negative numbers, e.g., on a number line, in counting, in temperature, "owing."

M1 b Demonstrates understanding of the base ten place value system and uses this knowledge to solve arithmetic tasks; that is:

- counts 1, 10, 100, or 1,000 more than or less than, e.g., 1 less than 10,000, 10 more than 380, 1,000 more than 23,000, 100 less than 9,000;
- uses knowledge about ones, tens, hundreds, and thousands to figure out answers to multiplication and division tasks, e.g., 36 x 10, 18 x 100, 7 x 1,000, 4,000 ÷ 4.

M1 c Estimates, approximates, rounds off, uses landmark numbers, or uses exact numbers, as appropriate, in calculations.

M1 d Describes and compares quantities by using concrete and real world models of simple fractions; that is:

- finds simple parts of wholes;
- recognizes simple fractions as instructions to divide, e.g., ¼ of something is the same as dividing something by 4;
- recognizes the place of fractions on number lines, e.g., in measurement;
- uses drawings, diagrams, or models to show what the numerator and denominator mean, including when adding like fractions, e.g., ⅛ + ⅝, or when showing that ¾ is more than ⅜;
- uses beginning proportional reasoning and simple ratios, e.g., "about half of the people."

M1 e Describes and compares quantities by using simple decimals; that is:

- adds, subtracts, multiplies, and divides money amounts;
- recognizes relationships among simple fractions, decimals, and percents, i.e., that ½ is the same as 0.5, and ½ is the same as 50%, with concrete materials, diagrams, and in real world situations, e.g., when discovering the chance of a coin landing on heads or tails.

M1 f Describes and compares quantities by using whole numbers up to 10,000; that is:

- connects ideas of quantities to the real world, e.g., how many people fit in the school's cafeteria; how far away is a kilometer;
- finds, identifies, and sorts numbers by their properties, e.g., odd, even, multiple, square.

Examples of activities through which students might demonstrate conceptual understanding of arithmetic and number include:

Use examples and drawings to show a third grader who is having trouble understanding multiplication why 3 x 6 = 6 x 3. **1a**

Use base ten blocks and numerals or other models and representations to solve 43 x 38, and show how the numbers get taken apart in the process of solving such a problem, e.g., 43 can become 40 + 3, or 38 can become 40 - 2. **1a, 1b**

Figure out how many Valentine's Day cards would be exchanged in total, if there are 30 students in the class and if everyone gave everyone else one card. **1a, 1b, 1c**

Draw and explain many different ways to make 263, using tens, hundreds and ones. **1a, 1b, 1c**

Organize a budget for a project. **1a, 1b, 1c, 1d, 1e, 1f, A1a, A1b, A1c**

Solve the following problem: "You have a book to read that is 100 pages long. You must read it in five days. How many pages should you read each day?" **1a, 1b, 1c, 1f**

Determine the possible number of objects in a group given this information: when put in groups of three, none is left over; when put in groups of two, one is left over; when put in groups of five, one is left over; there are fewer than 40 objects altogether. **1a, 1b, 1c, 1f**

Make reasonable estimates and then calculate accurately the number of beans in a cup, seeds in a pumpkin, ceiling tiles in a room, and wheels on 37 tricycles. **1a, 1b, 1c, 1f**

Show and explain to a classmate the different coin combinations that make 75¢, excluding pennies. **1a, 1b, 1c, 1f**

Find one possible answer for addends that equal the sum of 8,829, when the digits 1, 2, 3, 4, 5, 6, 7, and 8 are used only once in the solution. **1a, 1b, 1c, 5a, 5b, 5c**

Analyze seasonal variations in temperature, including negative values. **1a, 1c, 1f, S3c, A1a**

Put together fraction pieces to make a whole in different ways, e.g., ⅛ + ⅛ + ¾. **1c, 1d**

Draw diagrams to explain how three pizzas can be shared equally by four people. **1c, 1d**

Make approximate counts of different kinds of animals in different zones of a small tide pool area. **1c, 1f, S6b**

Draw and label diagrams to show the fractional value of each piece of a set of tangram pieces, if all seven pieces (i.e., the whole set) are equal to one whole. **1d**

Pretend you see a highway sign that reads, "Exit A, 1 mile. Exit B, 2¼ miles," and shortly thereafter you see a sign that reads, "Exit A, ¼ mile." Figure out how far away exit B must now be. (Balanced Assessment) **1d, 1f**

Make up stories that go with number sentences, e.g., 5 x 9 = 45; 27 x 6 = ?. **1f**

To see how these performance descriptions compare with the expectations for middle school and high school, turn to pages 220-231.

The examples that follow the performance descriptions for each standard are examples of the work students might do to demonstrate their achievement. The examples also indicate the nature and complexity of activities that are appropriate to expect of students at the elementary level. Depending on the nature of the task, the work might be done in class, for homework, or over an extended period.

The cross-references that follow the examples highlight examples for which the same activity, and possibly even the same piece of work, may enable students to demonstrate their achievement in relation to more than one standard. In some cases, the cross-references highlight examples of activities through which students might demonstrate their achievement in relation to standards for more than one subject matter.

M Geometry and Measurement Concepts

The student demonstrates understanding of a mathematical concept by using it to solve problems, representing it in multiple ways (through numbers, graphs, symbols, diagrams, or words, as appropriate), and explaining it to someone else. All three ways of demonstrating understanding—use, represent, and explain—are required to meet this standard.

The student produces evidence that demonstrates understanding of geometry and measurement concepts; that is, the student:

M2 a Gives and responds to directions about location, e.g., by using words such as "in front of," "right," and "above."

M2 b Visualizes and represents two dimensional views of simple rectangular three dimensional shapes, e.g., by showing the front view and side view of a building made of cubes.

M2 c Uses simple two dimensional coordinate systems to find locations on a map, and represent points and simple figures.

M2 d Uses many types of figures (angles, triangles, squares, rectangles, rhombi, parallelograms, quadrilaterals, polygons, prisms, pyramids, cubes, circles, and spheres) and identifies the figures by their properties, e.g., symmetry, number of faces, two- or three-dimensionality, no right angles.

M2 e Solves problems by showing relationships between and among figures, e.g., using congruence and similarity, and using transformations including flips, slides, and rotations.

M2 f Extends and creates geometric patterns using concrete and pictorial models.

M2 g Uses basic ways of estimating and measuring the size of figures and objects in the real world, including length, width, perimeter, and area.

M2 h Uses models to reason about the relationship between the perimeter and area of rectangles in simple situations.

M2 i Selects and uses units, both formal and informal as appropriate, for estimating and measuring quantities such as weight, length, area, volume, and time.

M2 j Carries out simple unit conversions, such as between cm and m, and between hours and minutes.

M2 k Uses scales in maps, and uses, measures, and creates scales for rectangular scale drawings based on work with concrete models and graph paper.

Examples of activities through which students might demonstrate conceptual understanding of geometry and measurement include:

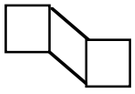
Locate a point on a map using its distance away from another given point, the map's scale, and compass point directions, e.g., N, NE, E, SE, etc. **2a, 2c, 2k**

Given views of three faces of a three dimensional figure made of stacked cubes, represent the views of the other faces and figure out the total number of cubes used to construct the figure. **2b**

Explain and show why a square is not a cube. **2d**

Describe a shape from among the following, using as few attributes as possible to distinguish it from the others: right triangle, equilateral triangle, trapezoid, and parallelogram. **2d**

Design and label a quilt square which includes two lines of symmetry, four congruent shapes, and two similar shapes; then make a final version at twice the original size, i.e., a "two to one" scale, and add color while maintaining symmetry. **2d, 2e, 2f, 2k**

Draw  at each of three consecutive 90° rotations in a clockwise direction. **2e**

Find all the shapes that can be made with five squares if the sides touch completely. **2e**

Figure out the approximate area and perimeter of the bottom of a shoe. **2g**

Solve the following problem, using concrete materials as appropriate: "Given a diagram of a fenced-in, rectangular garden plot with dimensions three meters by eight meters, find its area and perimeter; design a second garden plot using less fencing, but providing greater area; design a third plot using more fencing, but providing less area. Of all possible rectangular designs using the original amount of fencing, which provides the greatest area?" **2g, 2h, 2i**

Put five objects, such as books, rocks, or pumpkins, in rank order by weight, first by estimating and then by measuring exactly. **2i**

Figure out exactly how much time would you gain for the completion of a project by waking up at 6:15 rather than 7:00 for five school days. **2i, 2j**

Samples of student work that illustrate standard-setting performances for these standards can be found on pages 66-129.

Performance Descriptions *Mathematics*

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M Function and Algebra Concepts

The student demonstrates understanding of a mathematical concept by using it to solve problems, representing it in multiple ways (through numbers, graphs, symbols, diagrams, or words, as appropriate), and explaining it to someone else. All three ways of demonstrating understanding—use, represent, and explain—are required to meet this standard.

The student produces evidence that demonstrates understanding of function and algebra concepts; that is, the student:

M3 a Uses linear patterns to solve problems; that is:

- shows how one quantity determines another in a linear ("repeating") pattern, i.e., describes, extends, and recognizes the linear pattern by its rule, such as, the total number of legs on a given number of horses can be calculated by counting by fours;
- shows how one quantity determines another quantity in a functional relationship based on a linear pattern, e.g., for the "number of people and total number of eyes," figure out how many eyes 100 people have all together.

M3 b Builds iterations of simple non-linear patterns, including multiplicative and squaring patterns (e.g., "growing" patterns) with concrete materials, and recognizes that these patterns are not linear.

M3 c Uses the understanding that an equality relationship between two quantities remains the same as long as the same change is made to both quantities.

M3 d Uses letters, boxes, or other symbols to stand for any number, measured quantity, or object in simple situations with concrete materials, i.e., demonstrates understanding and use of a beginning concept of a variable.

Examples of activities through which students might demonstrate conceptual understanding of functions and algebra include:

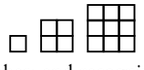
Find, make, and describe linear patterns on the 99-chart, e.g., 4, 14, 24, 34. **3a**

Given the situation described in the Christmas carol, "The Twelve Days of Christmas," determine how many gifts would have been given in total; describe any pattern you notice that helps in solving the problem. **3a, 3b**

Show how the letters "aab, aab,...," can represent the pattern "metal, metal, plastic,...," "leaf, leaf, rock,...," or many other patterns. **3a, 3d**

Solve the following problem: When building a staircase out of cubes, one step uses a total of one cube, two steps a total of three cubes, three steps a total of six cubes, etc. Find how many cubes are used for six steps, nine steps, n steps. Describe the pattern. **3b**

Observe and record, in a two-column table, multiplicative patterns with concrete materials, e.g., how many regions are produced by increasing the numbers of folds of paper; and recognize that this type of pattern does not proceed in a linear way, i.e., not 2, 4, 6, 8, etc., but 2, 4, 8, 16, etc. **3b**

Build the fourth, fifth, and sixth iterations in the following sequence:  ; use any given number to make a square number; and recognize that the number of little squares in the pattern does not proceed in a linear fashion. **3b**

Given x + y = 10 (or, say, + = 10), figure out all the whole numbers that will make the equation true, i.e., use the variables to show all the ways to make ten by adding two whole numbers together. **3c, 3d**

Plot points on a coordinate graph according to the convention that (x, y) refers to the intersection of a given vertical line and a given horizontal line. **3d**

M4 Statistics and Probability Concepts

The student demonstrates understanding of a mathematical concept by using it to solve problems, representing it in multiple ways (through numbers, graphs, symbols, diagrams, or words), and explaining it to someone else. All three ways of demonstrating understanding—use, represent, and explain—are required to meet this standard.

The student produces evidence that demonstrates understanding of statistics and probability concepts in the following areas; that is, the student:

M4 a Collects and organizes data to answer a question or test a hypothesis by comparing sets of data.

M4 b Displays data in line plots, graphs, tables, and charts.

M4 c Makes statements and draws simple conclusions based on data; that is:

- reads data in line plots, graphs, tables, and charts;
- compares data in order to make true statements, e.g., "seven plants grew at least 5 cm";
- identifies and uses the mode necessary for making true statements, e.g., "more people chose red";
- makes true statements based on a simple concept of average (median and mean), for a small sample size and where the situation is made evident with concrete materials or clear representations;
- interprets data to determine the reasonableness of statements about the data, e.g., "twice as often," "three times faster";
- uses data, including statements about the data, to make a simple concluding statement about a situation, e.g., "This kind of plant grows better near sunlight because the seven plants that were near the window grew at least 5 cm."

M4 d Gathers data about an entire group or by sampling group members to understand the concept of sample, i.e., that a large sample leads to more reliable information, e.g., when flipping coins.

M4 e Predicts results, analyzes data, and finds out why some results are more likely, less likely, or equally likely.

M4 f Finds all possible combinations and arrangements within certain constraints involving a limited number of variables.

Examples of activities through which students might demonstrate conceptual understanding of statistics and probability include:

Generate survey questions, such as: How many people are in your family? What is your hair color? What do you do at recess? Predict the survey results, carry out the survey, graph the data, and write true statements about the data. **4a, 4b, 4c, 4d**

Figure out a need, e.g., the librarian's purchase of new library books; and collect data in order to make a recommendation. **4a, 4b, 4c, 4d**

Make conjectures, after sampling the particular situation, about how many raisins are in a given box taken from a set of boxes or what colors of tiles are in a bag. **4a, 4b, 4c, 4d**

Investigate the temperature over the entire school year. **4a, 4b, 4c, 4d, S3c, A1a**

Investigate the possible and likely or unlikely outcomes when rolling two number cubes and recording the sums. **4d, 4e, 4f**

Design spinners with regions red, blue and green, according to the following instructions: Spinner #1: red will certainly win; Spinner #2: red cannot win; Spinner #3: red is likely to win; Spinner #4: red, blue and green are equally likely to win; Spinner #5: red or blue will probably win. **4e**

Find all of the different combinations possible using three ice cream flavors, two sauces, and one topping; then discover which would allow the most new combinations—an additional ice cream flavor or a new topping. (Balanced Assessment) **4f**

M Problem Solving and Reasoning

The student demonstrates logical reasoning throughout work in mathematics, i.e., concepts and skills, problem solving, and projects; demonstrates problem solving by using mathematical concepts and skills to solve non-routine problems that do not lay out specific and detailed steps to follow; and solves problems that make demands on all three aspects of the solution process—formulation, implementation, and conclusion.

Formulation

M5 a Given the basic statement of a problem situation, the student:

- makes the important decisions about the approach, materials, and strategies to use, i.e., does not merely fill in a given chart, use a pre-specified manipulative, or go through a predetermined set of steps;
- uses previously learned strategies, skills, knowledge, and concepts to make decisions;
- uses strategies, such as using manipulatives or drawing sketches, to model problems.

Implementation

M5 b The student makes the basic choices involved in planning and carrying out a solution; that is, the student:

- makes up and uses a variety of strategies and approaches to solving problems and uses or learns approaches that other people use, as appropriate;
- makes connections among concepts in order to solve problems;
- solves problems in ways that make sense and explains why these ways make sense, e.g., defends the reasoning, explains the solution.

Conclusion

M5 c The student moves beyond a particular problem by making connections, extensions, and/or generalizations; for example, the student:

- explains a pattern that can be used in similar situations;
- explains how the problem is similar to other problems he or she has solved;
- explains how the mathematics used in the problem is like other concepts in mathematics;
- explains how the problem solution can be applied to other school subjects and in real world situations;
- makes the solution into a general rule that applies to other circumstances.

Examples of activities through which students might demonstrate facility with problem solving and reasoning include:

Suppose you are given a string that is sixteen inches long. If you cut or fold it in any two places, will it always make a triangle? **5a, 5b, 5c**

Figure out how many handshakes there will be altogether if five people in a room shake each other's hand just once. **5a, 5b, 5c**

Prove or disprove a classmate's claim that ¾ and ⅝ are really the same because both have one piece missing. **5a, 5b, 5c**

Given an unopened bag of chocolate chip cookies, with cookies arranged in rows, show and explain whether the manufacturer's claim of "More than 1,000 chips in every bag!" is reasonable. **5a, 5b, 5c**

Figure out the value of each tangram piece if the small tangram triangle is worth one cent and the value is proportional to the area. **5a, 5b, 5c**

For the sum of 8,829, find one possible answer for addends when the digits 1, 2, 3, 4, 5, 6, 7, and 8 are used only once in the solution. **5a, 5b, 5c**

Samples of student work that illustrate standard-setting performances for these standards can be found on pages 66-129.

Performance Descriptions *Mathematics*

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M Mathematical Skills and Tools

The student demonstrates fluency with basic and important skills by using these skills accurately and automatically, and demonstrates practical competence and persistence with other skills by using them effectively to accomplish a task, perhaps referring to notes, books, or other students, perhaps working to reconstruct a method; that is, the student:

M6 a Adds, subtracts, multiplies, and divides whole numbers correctly; that is:

- knows single digit addition, subtraction, multiplication, and division facts;
- adds and subtracts numbers with several digits;
- multiplies and divides numbers with one or two digits;
- multiplies and divides three digit numbers by one digit numbers.

M6 b Estimates numerically and spatially.

M6 c Measures length, area, perimeter, circumference, diameter, height, weight, and volume accurately in both the customary and metric systems.

M6 d Computes time (in hours and minutes) and money (in dollars and cents).

M6 e Refers to geometric shapes and terms correctly with concrete objects or drawings, including triangle, square, rectangle, side, edge, face, cube, point, line, perimeter, area, and circle; and refers with assistance to rhombus, parallelogram, quadrilateral, polygon, polyhedron, angle, vertex, volume, diameter, circumference, sphere, prism, and pyramid.

M6 f Uses +, -, x, ÷, /, $\sqrt{\ }$, $, ¢, %, and . (decimal point) correctly in number sentences and expressions.

M6 g Reads, creates, and represents data on line plots, charts, tables, diagrams, bar graphs, simple circle graphs, and coordinate graphs.

M6 h Uses recall, mental computations, pencil and paper, measuring devices, mathematics texts, manipulatives, calculators, computers, and advice from peers, as appropriate, to achieve solutions; that is, uses measuring devices, graded appropriately for given situations, such as rulers (customary to the ⅛ inch; metric to the millimeter), graph paper (customary to the inch or half-inch; metric to the centimeter), measuring cups (customary to the ounce; metric to the milliliter), and scales (customary to the pound or ounce; metric to the kilogram or gram).

Examples of activities through which students might demonstrate facility with mathematical skills and tools include:

Know that 6 x 7 = 42. **6a**

Use an efficient mental approach to estimate the total cost of items costing $2.94, $1.28 and $0.74. **6a, 6b, 6d, 6h**

Mentally add two digit numbers correctly during problem solving. **6a, 6h**

Decide whether to use a calculator, paper and pencil or mental arithmetic to figure out 6 x 6,000. **6a, 6h**

Figure out rectangular areas correctly when designing a floor plan for a "dream house." **6b, 6c**

Measure the circumference of a pumpkin accurately by using a piece of yarn, and then laying it next to a ruler. **6c, 6e**

Use a calculator to check the arithmetic in a project.

Use a table to record functions such as how many chairs fit at how many tables. **6g**

Use a Venn diagram to record students who wore a sweater to school and students who walked to school. **6g**

Make a bar graph or simple circle graph to show how many students like different kinds of vegetables. **6g, 6f**

M Mathematical Communication

The student uses the language of mathematics, its symbols, notation, graphs, and expressions, to communicate through reading, writing, speaking, and listening, and communicates about mathematics by describing mathematical ideas and concepts and explaining reasoning and results; that is, the student:

M7 a Uses appropriate mathematical terms, vocabulary, and language, based on prior conceptual work.

M7 b Shows mathematical ideas in a variety of ways, including words, numbers, symbols, pictures, charts, graphs, tables, diagrams, and models.

M7 c Explains solutions to problems clearly and logically, and supports solutions with evidence, in both oral and written work.

M7 d Considers purpose and audience when communicating about mathematics.

M7 e Comprehends mathematics from reading assignments and from other sources.

Examples of activities through which students might demonstrate facility with mathematical communication include:

Use words, numbers, or diagrams to explain how to take numbers apart in order to solve problems using mental math, e.g., 25 x 6. One way is "20 x 6 is 120, and 5 x 6 is 30; 120 + 30 = 150." Or, "25 x 4 is 100, and 25 x 2 is 50, and so the answer is 150 because 100 + 50 = 150." **7a, 7b, 7c**

Explain why 34 + 17 3417 to a first grader or to a visitor from outer space. **7a, 7b, 7c, 7d**

Show ½ + ¾ in pictures and diagrams so a younger student could understand the sum. **7a, 7b, 7c, 7d**

Use clear, correct mathematical language to describe a shape composed of several geometric solids so that it could be reproduced exactly by a person who cannot see the shape. **7a, 7b, 7c, 7d**

Represent survey data about student school lunch preferences in graphical, written, and numerical form in order to make a clear and effective recommendation to kitchen staff about menu changes. **7a, 7b, 7c, 7d**

Give an oral presentation of a preliminary investigation of classification of shapes, in order to get peer feedback; then revise the classification scheme to make it clearer. **7a, 7b, 7c, 7d, E3c**

Prepare a report, including graphs, charts, and diagrams, on the optimal number and location of recycling containers, based on data from the classroom and the entire school. **7a, 7b, 7c, 7d, 7e, E2a, S4b, S6d, S7a, A1b**

M Putting Mathematics to Work

The student conducts at least one large scale project each year, beginning in fourth grade, drawn from the following kinds and, over the course of elementary school, conducts projects drawn from at least two of the kinds.

A single project may draw on more than one kind.

M8 a Data study, in which the student:

- develops a question and a hypothesis in a situation where data could help make a decision or recommendation;
- decides on a group or groups to be sampled and makes predictions of the results, with specific percents, fractions, or numbers;
- collects, represents, and displays data in order to help make the decision or recommendation; compares the results with the predictions;
- writes a report that includes recommendations supported by diagrams, charts, and graphs, and acknowledges assistance received from parents, peers, and teachers.

M8 b Science study, in which the student:

- decides on a specific science question to study and identifies the mathematics that will be used, e.g., measurement;
- develops a prediction (a hypothesis) and develops procedures to test the hypothesis;
- collects and records data, represents and displays data, and compares results with predictions;
- writes a report that compares the results with the hypothesis; supports the results with diagrams, charts, and graphs; acknowledges assistance received from parents, peers, and teachers.

M8 c Design of a physical structure, in which the student:

- decides on a structure to design, the size and budget constraints, and the scale of design;
- makes a first draft of the design, and revises and improves the design in response to input from peers and teachers;
- makes a final draft and report of the design, drawn and written so that another person could make the structure; acknowledges assistance received from parents, peers, and teachers.

M8 d Management and planning, in which the student:

- decides on what to manage or plan, and the criteria to be used to see if the plan worked;
- identifies unexpected events that could disrupt the plan and further plans for such contingencies;
- identifies resources needed, e.g., materials, money, time, space, and other people;
- writes a detailed plan and revises and improves the plan in response to feedback from peers and teachers;
- carries out the plan (optional);
- writes a report on the plan that includes resources, budget, and schedule, and acknowledges assistance received from parents, peers, and teachers.

M8 e Pure mathematics investigation, in which the student:

- decides on the area of mathematics to investigate, e.g., numbers, shapes, patterns;
- describes a question or concept to investigate;
- decides on representations that will be used, e.g., numbers, symbols, diagrams, shapes, or physical models;
- carries out the investigation;
- writes a report that includes any generalizations drawn from the investigation, and acknowledges assistance received from parents, peers, and teachers.

Examples of projects include:

Develop questions and a hypothesis for a study of students' diets; collect, organize, display, and analyze the data; and make recommendations to the school community based on the data. **8a, S2a, S4c, A1b**

Compare the growth of a set of plants under a variety of conditions, e.g., amount of water, fertilizer, duration and exposure to sunlight. **8b, S2a**

Make a design for a tree house that accounts for physical and financial constraints. **8c, A1a**

Plan a class camping trip, including making a schedule, researching costs and facilities, developing a budget. **8d, S2a, S2c, S4b, S4c, A1c**

Plan and conduct a probability study that compares the results from three different spinners, e.g.,

. **8e**

Samples of student work that illustrate standard-setting performances for these standards can be found on pages 66-129.

Work Sample & Commentary: *Sharing 25*

Arithmetic and Number Concepts M1
Geometry & Measurement Concepts
Function & Algebra Concepts
Statistics & Probability Concepts
Problem Solving & Reasoning
Mathematical Skills & Tools M6
Mathematical Communication
Putting Mathematics to Work

The quotations from the Mathematics performance descriptions in this commentary are excerpted. The complete performance descriptions are shown on pages 60-65.

The task

The following written prompt appeared on an examination:

In each situation below, four friends want to "share 25" as equally as possible. Show or explain how to "share 25" in each situation.

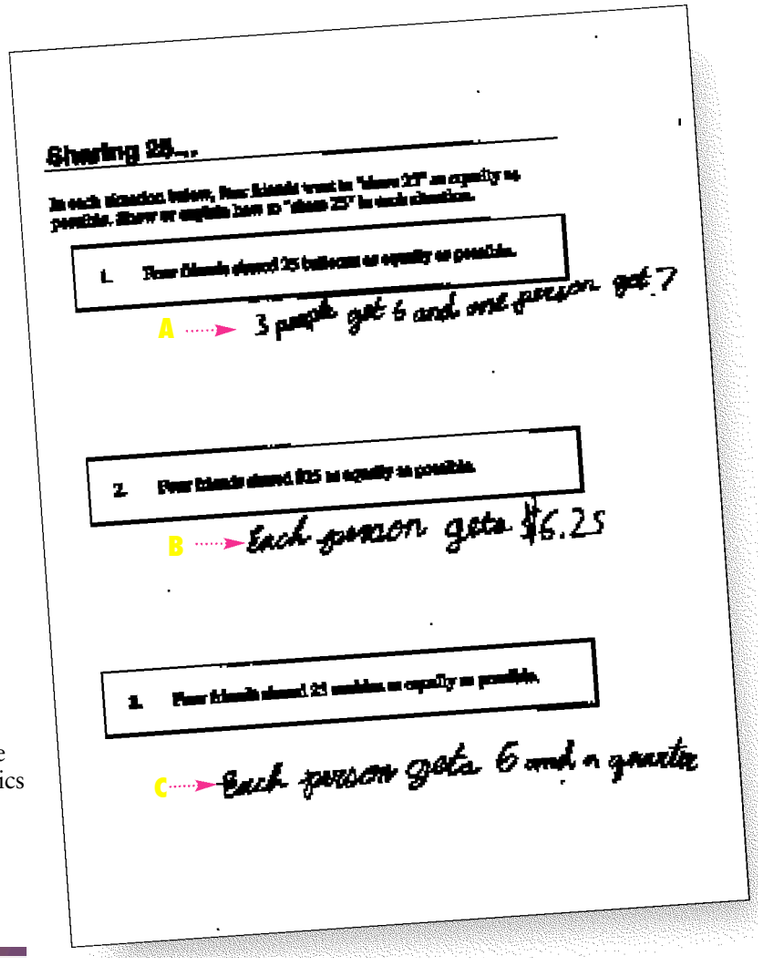
1. Four friends shared 25 balloons as equally as possible.
2. Four friends shared $25 as equally as possible.
3. Four friends shared 25 cookies as equally as possible.

Circumstances of performance

This sample of student work was produced under the following conditions:

alone	in a group
in class	as homework
with teacher feedback	with peer feedback
timed	opportunity for revision

This five minute task was part of a field test for the New Standards Reference Examination: Mathematics (Elementary).


Sharing 25...

In each situation below, Four friends want to "share 25" as equally as possible. Show or explain how to "share 25" in each situation.

1. Four friends shared 25 balloons as equally as possible.

A → 3 people get 6 and one person get 7

2. Four friends shared $25 as equally as possible.

B → Each person gets $6.25

3. Four friends shared 25 cookies as equally as possible.

C → Each person gets 6 and a quarter

This work sample illustrates a standard-setting performance for the following parts of the standards:

- **M1 a Arithmetic and Number Concepts: Add, subtract, multiply, and divide whole numbers.**
- **M1 c Arithmetic and Number Concepts: Estimate, approximate, round off, use landmark numbers, or use exact numbers in calculations.**
- **M1 d Arithmetic and Number Concepts: Describe and compare quantities by using simple fractions.**
- **M1 e Arithmetic and Number Concepts: Describe and compare quantities by using simple decimals.**
- **M6 a Mathematical Skills and Tools: Add, subtract, multiply, and divide whole numbers correctly.**
- **M6 d Mathematical Skills and Tools: Compute time and money.**
- **M6 f Mathematical Skills and Tools: Use +, -, x, ÷, /, √, $, ¢, %, and . (decimal point) correctly in number sentences and expressions.**

What the work shows

M1 a Arithmetic and Number Concepts: The student adds, subtracts, multiplies, and divides whole numbers, with and without calculators; that is:

- **divides, i.e., puts things into groups, shares equally….**

A B C

- **analyzes problem situations and contexts in order to figure out when to add, subtract, multiply, or divide.**

A B C The student demonstrated conceptual understanding by applying arithmetic skills differently, and appropriately, in a variety of situations.

Sharing 25

M1 c Arithmetic and Number Concepts: The student estimates, approximates, rounds off, uses landmark numbers, or uses exact numbers, as appropriate, in calculations.

A B C The correct answers demonstrate rounding off or use of exact number, as appropriate, in each situation.

M1 d Arithmetic and Number Concepts: The student describes and compares quantities by using concrete and real world models of simple fractions; that is:

• finds simple parts of wholes….

B C

M1 e Arithmetic and Number Concepts: The student describes and compares quantities by using simple decimals; that is:

• adds, subtracts, multiplies, and divides money amounts….

B

M6 a Mathematical Skills and Tools: The student adds, subtracts, multiplies, and divides whole numbers correctly….

A B C

M6 d Mathematical Skills and Tools: The student computes…money (in dollars and cents).

B

M6 f Mathematical Skills and Tools: The student uses…$…and . (decimal point) correctly….

B

Work Sample & Commentary: *Arithmetic*

Arithmetic and Number Concepts M1
Geometry & Measurement Concepts
Function & Algebra Concepts
Statistics & Probability Concepts
Problem Solving & Reasoning
Mathematical Skills & Tools M6
Mathematical Communication M7
Putting Mathematics to Work

The quotations from the Mathematics performance descriptions in this commentary are excerpted. The complete performance descriptions are shown on pages 60-65.

The tasks

These tasks all focus on the same arithmetic and number concepts and skills, although they were drawn from four different classrooms.

In Sample 1, the teacher included the following instructions on a classroom test:

Using what you know about multiplication, find the answer to the equation below. Show all your work and clearly explain how you got your answer.

63 x 46 =

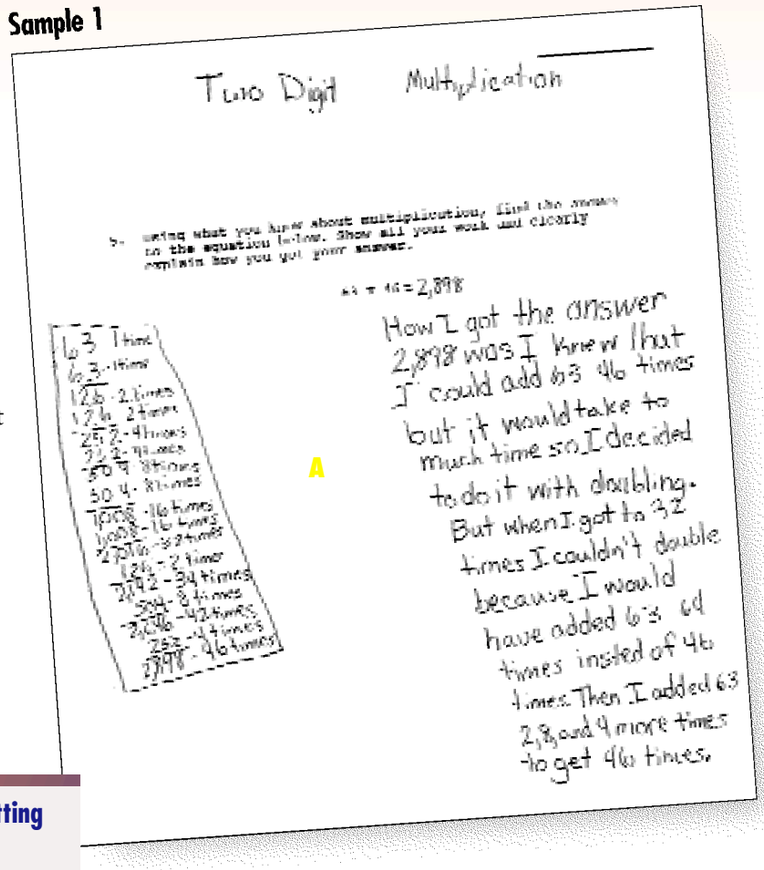
In Sample 2, the teacher gave the following written instructions as part of a class exercise:

Solve each problem two ways.

522 - 367 =

87 x 9 =

Sample 1



These work samples illustrate standard-setting performances for the following parts of the standards:

- **M1 a Arithmetic and Number Concepts: Add, subtract, multiply, and divide whole numbers.**
- **M1 b Arithmetic and Number Concepts: Demonstrate understanding of the base ten value system and use this knowledge to solve arithmetic tasks.**
- **M1 c Arithmetic and Number Concepts: Estimate, approximate, round off, use landmark numbers, or use exact numbers in calculations.**
- **M1 d Arithmetic and Number Concepts: Describe and compare quantities by using simple fractions.**
- **M6 a Mathematical Skills and Tools: Add, subtract, multiply, and divide whole numbers correctly.**
- **M6 f Mathematical Skills and Tools: Use +, -, x, ÷, /, $\sqrt{\ }$, $, ¢, %, and . (decimal point) correctly in number sentences and expressions.**
- **M6 h Mathematical Skills and Tools: Use recall, mental computations, and pencil and paper to achieve solutions.**
- **M7 a Mathematical Communication: Use appropriate mathematical terms, vocabulary, and language.**
- **M7 b Mathematical Communication: Show mathematical ideas in a variety of ways.**
- **M7 c Mathematical Communication: Explain solutions to problems clearly and logically.**

In Sample 3, the teacher gave the following oral instructions at the end of a long unit of work:

Make a catalogue of all the ways the class has come up with to multiply large numbers.

Make up a two digit by two digit multiplication problem and use these ways to find the answer.

In Sample 4, a specialist read the following written instructions to an individual student:

Four children had three bags of M & M's.

They decided to open all three bags of candy and share the M & M's fairly.

There were 52 M & M candies in each bag.

How many M & M candies did each child get?

Circumstances of performance

These samples of student work were produced under the following conditions:

alone	in a group
in class	as homework
with teacher feedback	with peer feedback
timed	opportunity for revision

Sample 1 was part of a classroom test. For Samples 2 and 3, students were permitted to talk as they worked. Sample 4 was produced when a specialist interviewed the student as the student was completing the work.

Arithmetic

Sample 1 was produced before the class had received any instruction about two digit by two digit multiplication, although they had experience in developing strategies for simpler computation.

Sample 2 was produced in a classroom where students' computation strategies were regularly shared, and several were posted on the wall.

Sample 3 was produced after a long unit of work in which students created, developed, and learned a range of strategies for single and double digit multiplication.

Sample 4 was produced before the class had received any instruction about division, but they had extensive experience with developing and explaining strategies for other computation problems.

What the work shows

M1 a Arithmetic and Number Concepts: The student adds, subtracts, multiplies, and divides whole numbers, with and without calculators; that is:

- **adds, i.e., joins things together, increases.**

A B C D E All students added as they solved larger problems.

- **subtracts, i.e., takes away, compares, finds the difference.**

F G

- **multiplies, i.e., uses repeated addition, counts by multiples, combines things that come in groups, makes arrays….**

A The student combined doubling with repeated addition.

C The student used repeated addition of 85.

H The student showed how the problem could be solved by making 62 circles with 85 stars in each circle.

I The student drew a base ten block array.

- **divides, i.e., puts things into groups, shares equally….**

J The student "split" 80 into 40 and 40 in order to multiply.

E K The student began to divide 156 by 4 by putting down 20 four times.

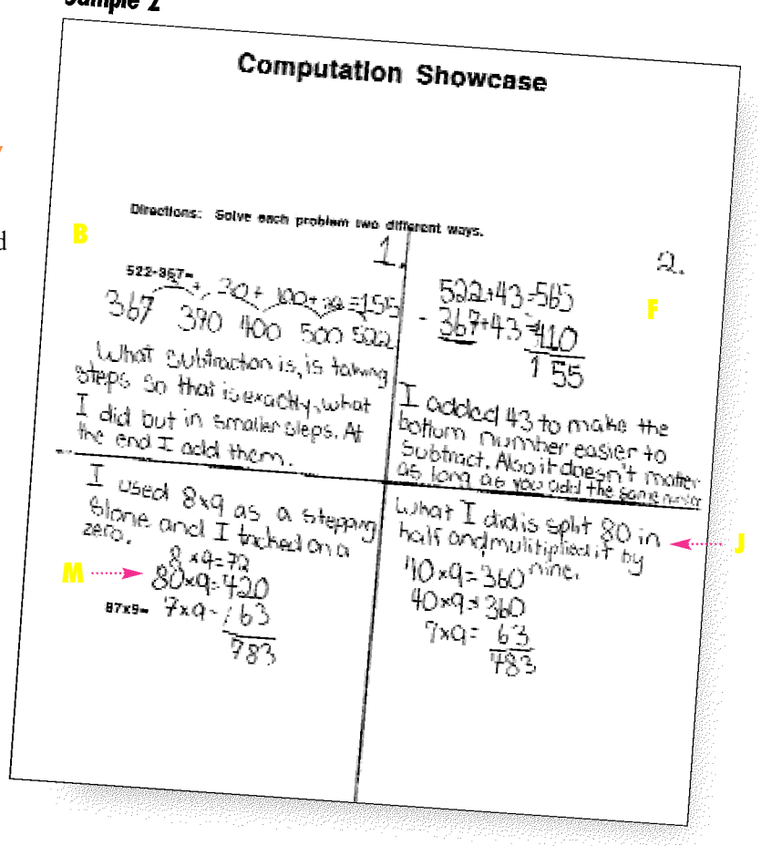
- **analyzes problem situations and contexts in order to figure out when to add, subtract, multiply, or divide.**

E In Sample 4, the instructions give no indication about how arithmetic can be used to solve the problem: the student figured it all out.

- **solves arithmetic problems by relating addition, subtraction, multiplication, and division to one another.**

A C The students related addition to multiplication.

Sample 2



Arithmetic

Arithmetic and Number Concepts M1
Geometry & Measurement Concepts
Function & Algebra Concepts
Statistics & Probability Concepts
Problem Solving & Reasoning
Mathematical Skills & Tools M6
Mathematical Communication M7
Putting Mathematics to Work

B F These parts of the work demonstrate using addition to solve a subtraction problem.

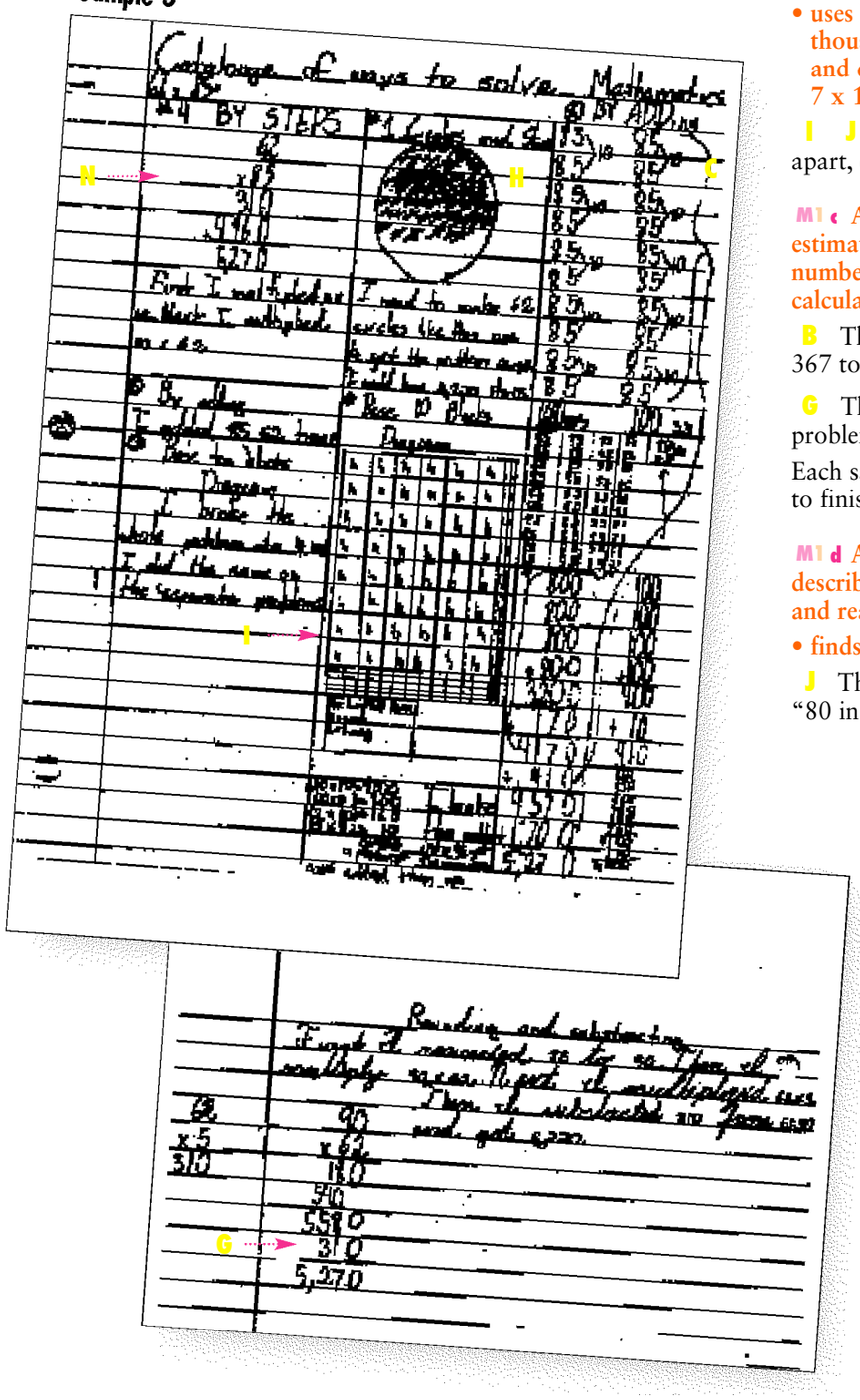
L The student counted by twos in the process of multiplying.

D The student counted by multiples to solve a division problem.

- computes answers mentally, e.g., 27 + 45, 30 x 4.

A B C E All students used mental computation effectively throughout all of their work.

Sample 3



M1 b Arithmetic and Number Concepts: The student demonstrates understanding of the base ten place value system and uses this knowledge to solve arithmetic tasks; that is:

- counts 1, 10, 100, or 1,000 more than or less than, e.g., 1 less than 10,000, 10 more than 380, 1,000 more than 23,000, 100 less than 9,000.

B The student counted 3, 30, and 100 more.

D The student counted by twenties, tens, and ones.

Each sample includes evidence that demonstrates this kind of understanding.

- uses knowledge about ones, tens, hundreds, and thousands to figure out answers to multiplication and division tasks, e.g., 36 x 10, 18 x 100, 7 x 1,000, 4,000 ÷ 4.

I J L M The students broke the numbers apart, e.g., 40 x 9; 60 x 80; 3 x 50.

M1 c Arithmetic and Number Concepts: The student estimates, approximates, rounds off, uses landmark numbers, or uses exact numbers, as appropriate, in calculations.

B The student used landmark numbers to go from 367 to 370 to 400 to 500.

G The student rounded off in order to make the problem easier.

Each sample demonstrates the use of exact numbers to finish the tasks.

M1 d Arithmetic and Number Concepts: The student describes and compares quantities by using concrete and real world models of simple fractions; that is:

- finds simple parts of wholes.

J The student applied concrete knowledge to split "80 in half."

Arithmetic

M6 a Mathematical Skills and Tools: The student adds, subtracts, multiplies, and divides whole numbers correctly; that is:

- **knows single digit addition, subtraction, multiplication, and division facts.**

A C L M Here as in many places, the students' knowledge of basic facts is demonstrated in correct solutions to larger problems.

- **adds and subtracts numbers with several digits.**

A B C E Here, as in other places, the samples demonstrate correct work.

- **multiplies and divides numbers with one or two digits.**

A J M N Here, as in many other places, the samples demonstrate correct work.

- **multiplies and divides three digit numbers by one digit numbers.**

E In elementary school, it may be appropriate for students to work through a solution to division problems; multiplication and division with multi-digit numbers is not expected to be routine at this level.

M6 f Mathematical Skills and Tools: The student uses +, -, x…correctly in number sentences and expressions.

F I These parts of the samples demonstrate correct use of symbols. This is also evident throughout the samples.

M6 h Mathematical Skills and Tools: The student uses recall, mental computations, pencil and paper…to achieve solutions.

A B C E

M7 a Mathematical Communication: The student uses appropriate mathematical terms, vocabulary, and language, based on prior conceptual work.

A B C E

M7 b Mathematical Communication: The student shows mathematical ideas in a variety of ways, including words, numbers, symbols, pictures, charts, graphs, tables, diagrams, and models.

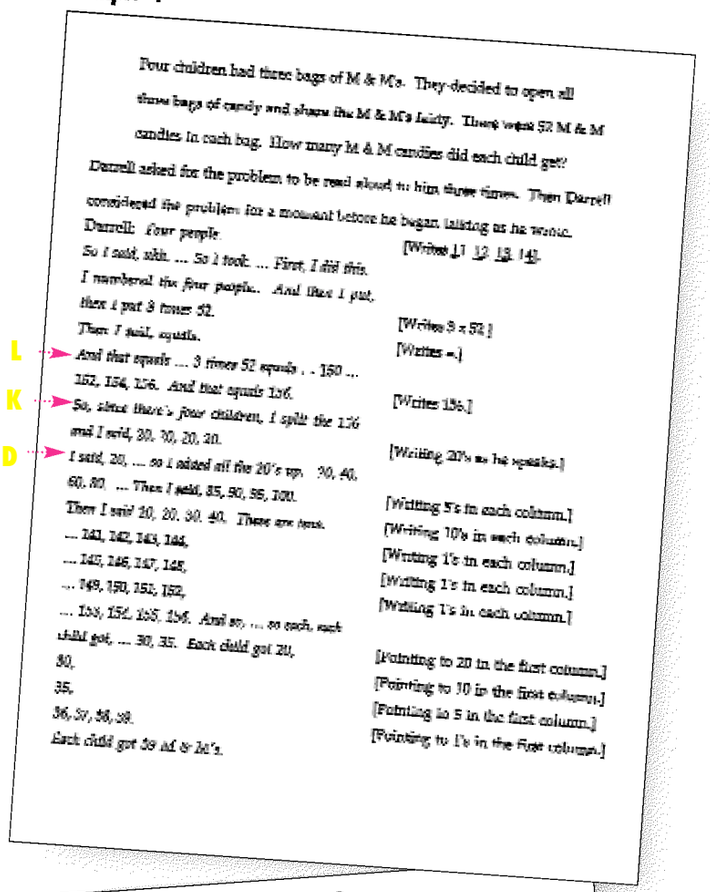
A B C E

M7 c Mathematical Communication: The student explains solutions to problems clearly and logically, and supports solutions with evidence, in both oral and written work.

A B D G

In Sample 3, the student spelled a few words inconsistently, e.g., "multiplied" is correct while "multiplyed" is not; and misspelled others, e.g., "Catalouge" instead of "Catalogue." All of the samples were taken from work done in class which was not edited for spelling.

Sample 4



Four children had three bags of M & M's. They decided to open all three bags of candy and share the M & M's fairly. There were 52 M & M candies in each bag. How many M & M candies did each child get?

Darrell asked for the problem to be read aloud to him three times. Then Darrell considered the problem for a moment before he began talking as he wrote.

Darrell: *Four people* [Writes 1, 2, 3, 4.]

So I said, ahh. … So I took. … First, I did this. I numbered the four people. And then I put, then I put 3 times 52. [Writes 3 x 52.]

Then I said, equals. [Writes =.]

L *And that equals … 3 times 52 equals . . 150 … 152, 154, 156. And that equals 156.* [Writes 156.]

K *So, since there's four children, I split the 156 and I said, 30, 30, 20, 20.* [Writing 20's as he speaks.]

D *I said, 20, … so I added all the 20's up. 20, 40, 60, 80 … Then I said, 35, 90, 95, 100.* [Writing 5's in each column.]

Then I said 10, 20, 30, 40. There are tens. [Writing 10's in each column.]

… 141, 142, 143, 144, [Writing 1's in each column.]

… 145, 146, 147, 148, [Writing 1's in each column.]

… 149, 150, 151, 152, [Writing 1's in each column.]

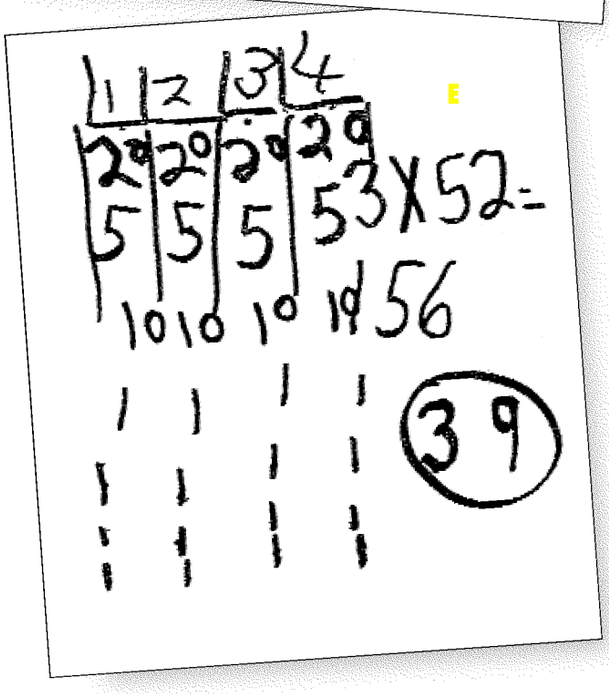
… 153, 154, 155, 156. And so, … so each, each child got, … 30, 35. Each child got 20, [Pointing to 20 in the first column.]

30, [Pointing to 10 in the first column.]

35, [Pointing to 5 in the first column.]

36, 37, 38, 39. [Pointing to 1's in the first column.]

Each child got 39 M & M's.



Work Sample & Commentary: *3-D to 2-D*

The quotations from the Mathematics performance descriptions in this commentary are excerpted. The complete performance descriptions are shown on pages 60-65.

The tasks

These work samples are drawn from two different classrooms, but focus on the same concept of visualizing and representing three dimensional objects in two dimensions.

In Sample 1, the teacher gave the following written instruction to the students:

Andre built a shape with blocks. What would be the front view?

In Sample 2, the teacher gave the students Student Sheets 15 and 16 (from *Seeing Solids and Silhouettes*, a book in the series, *Investigations in Number, Data and Space*, Dale Seymour Publications, 1995) which had these written instructions:

Choose two or three buildings. Draw their silhouettes from the front, top, and right side. (Student Sheet 15)

Draw the front, top, and side silhouettes for three cube buildings. Put the number of the building above its silhouettes. (Student Sheet 16)

Circumstances of performance

These samples of student work were produced under the following conditions:

alone	in a group
in class	as homework
with teacher feedback	with peer feedback
timed	opportunity for revision

Both students were developing a New Standards Elementary Mathematics Portfolio. Because of this, they knew the criteria in this system for work that shows conceptual understanding. The criteria require the student to use, represent, and explain the concept. The students used the tasks as opportunities to show what they understood about the concept of visualizing and representing three dimensional objects in two dimensions.

Sample 1 was completed in Spanish in a bilingual classroom. The translation was provided by the teacher.

What the work shows

M2 a Geometry and Measurement Concepts: The student gives and responds to directions about location, e.g., by using words such as "in front of," "right," and "above."

A B Both students used location words in their explanations of their thinking and understanding.

M2 b Geometry and Measurement Concepts: The student visualizes and represents two dimensional views of simple rectangular three dimensional shapes, e.g., by showing the front view and side view of a building made of cubes.

C D Both students produced two dimensional views to represent three dimensional objects.

E Sample 2 includes a small mistake that does not detract from the evidence of an overall understanding of this concept.

These work samples illustrate standard-setting performances for the following parts of the standards:

M2 a Geometry and Measurement Concepts: Give and respond to directions about location.

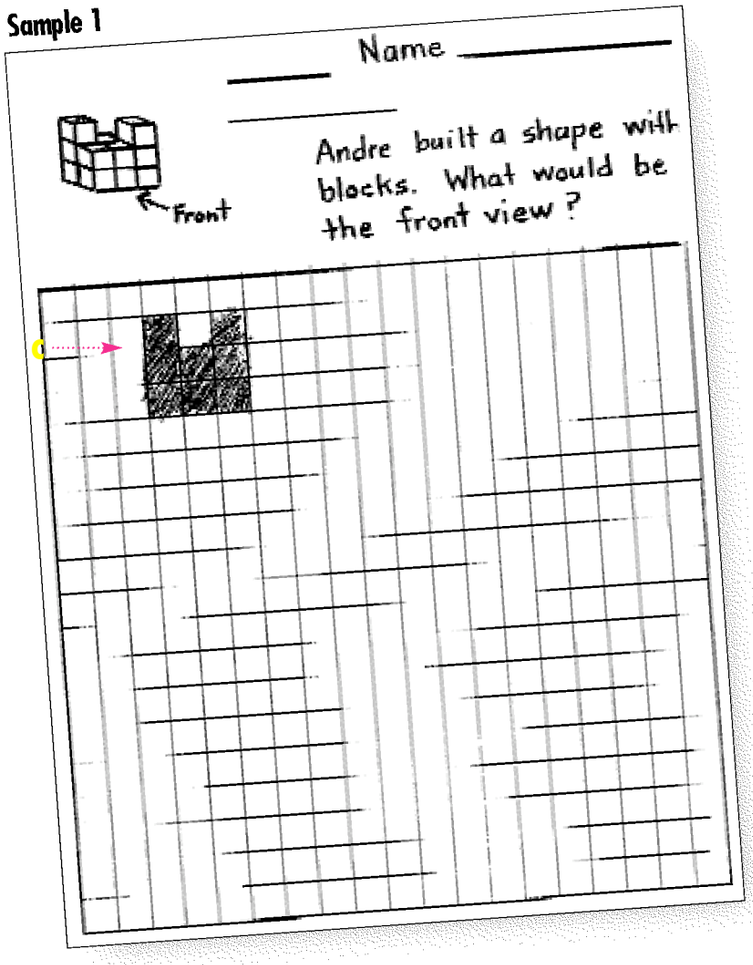
M2 b Geometry and Measurement Concepts: Visualize and represent two dimensional views of simple rectangular three dimensional shapes.

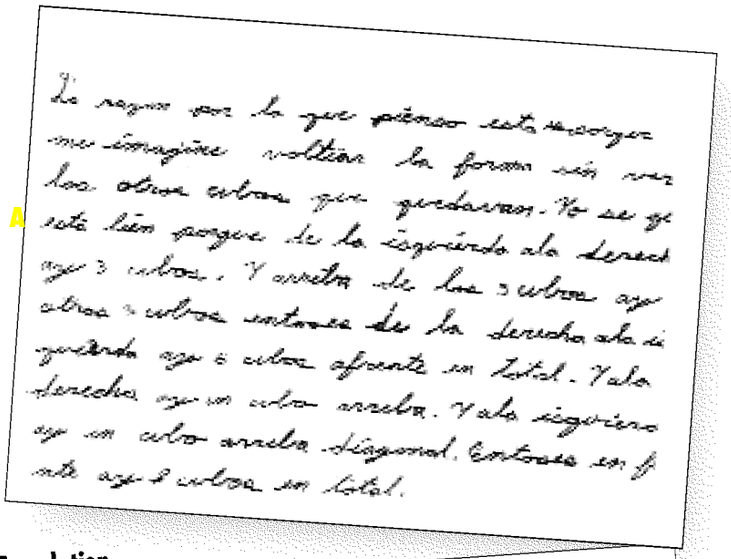
M2 d Geometry and Measurement Concepts: Use many types of figures.

M7 a Mathematical Communication: Use appropriate mathematical terms, vocabulary, and language.

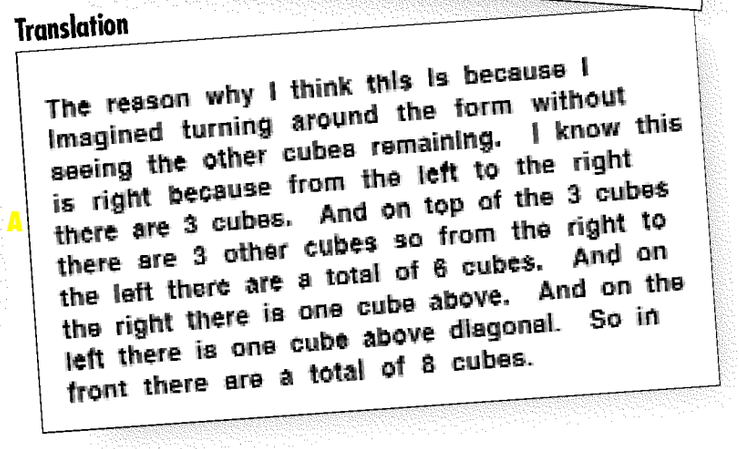
3-D to 2-D

Sample 1





Translation



The reason why I think this is because I imagined turning around the form without seeing the other cubes remaining. I know this is right because from the left to the right there are 3 cubes. And on top of the 3 cubes there are 3 other cubes so from the right to the left there are a total of 6 cubes. And on the right there is one cube above. And on the left there is one cube above diagonal. So in front there are a total of 8 cubes.

M2 d Geometry and Measurement Concepts: The student uses many types of figures (…squares,…cubes…) and identifies the figures by their properties, e.g., symmetry, number of faces, two- or three-dimensionality, no right angles.

A B C D

3-D to 2-D

Sample 2

March 8, 96

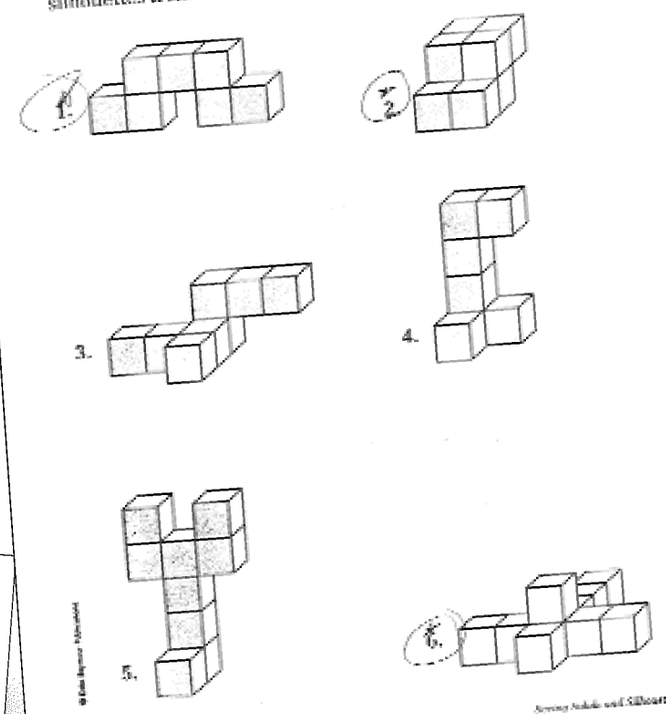
F Spatial relationships are really interesting, fun, and exciting. I built these shapes with intro-looking cubes and looked at things from the front, top, and side. This is called perspective. B Perspective means looking at things from different view or angle.

for example

Student Sheet 15

More Cube Buildings

Choose two or three buildings. Draw their silhouettes from the front, top, and right side.



Seeing Solids and Silhouettes • 97

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D

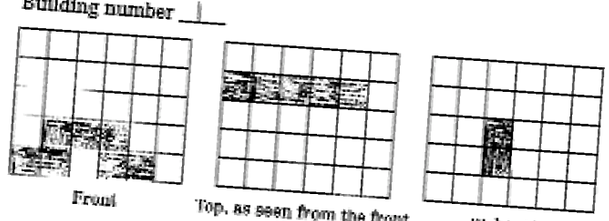
Date

Student Sheet 16

More Cube Silhouettes

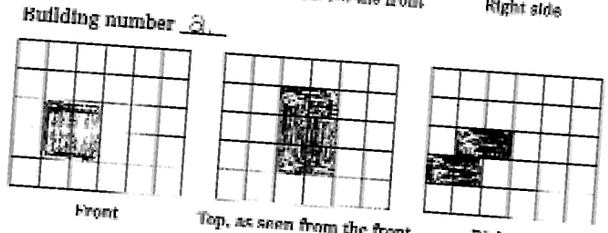
Draw the front, top, and side silhouettes for three cube buildings. Put the number of the building above its silhouettes.

Building number 1



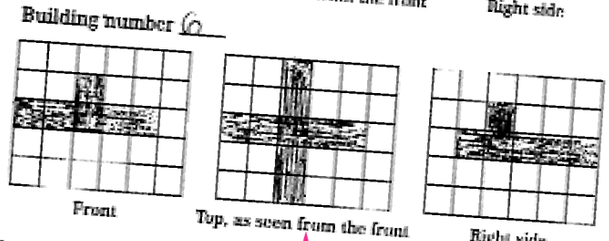
Front | Top, as seen from the front | Right side

Building number 3



Front | Top, as seen from the front | Right side

Building number 6



Front | Top, as seen from the front | Right side

E

98 • Seeing Solids and Silhouettes

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M7 a Mathematical Communication: The student uses appropriate mathematical terms, vocabulary, and language, based on prior conceptual work.

A Sample 1 includes appropriate use of location words (e.g., "from the right to the left") and shape words ("cube").

B Sample 2 includes a good elementary definition of perspective, supported with a clearly explained example.

F Sample 2 includes some unnecessary and superfluous communication. While this kind of communication is fairly common for students as they develop their capacity to explain their thinking, reasoning, and understanding, it does not add any mathematical communication to the response.

Arithmetic and Number Concepts
Geometry & Measurement Concepts M2
Function & Algebra Concepts
Statistics & Probability Concepts
Problem Solving & Reasoning
Mathematical Skills & Tools
Mathematical Communication M7
Putting Mathematics to Work

Work Sample & Commentary: *Patterns*

The task

The teacher asked students to respond in their mathematics journals to the following question: What is a mathematical pattern?

Circumstances of performance

This sample of student work was produced under the following conditions:

alone	in a group
in class	as homework
with teacher feedback	with peer feedback
timed	opportunity for revision

What the work shows

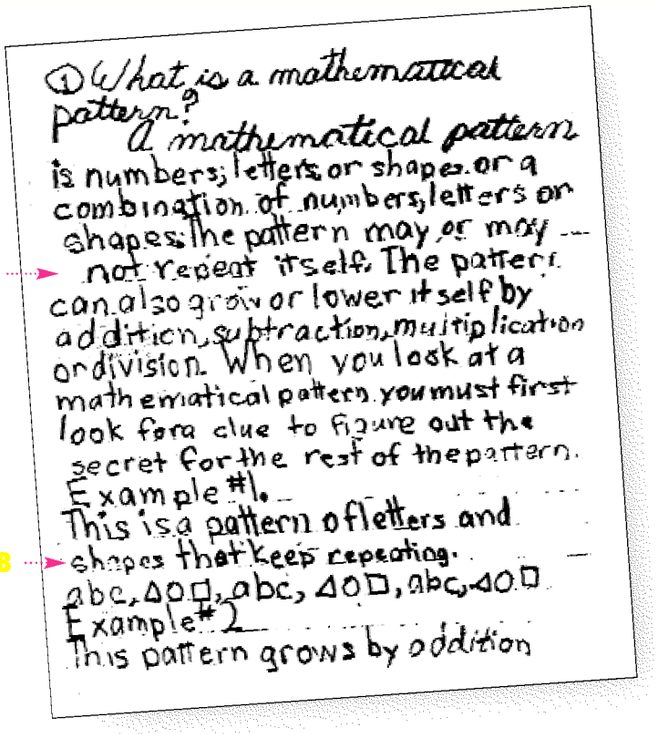
M3 a Function and Algebra Concepts: The student uses linear patterns to solve problems; that is, shows how one quantity determines another in a linear ("repeating") pattern, i.e., describes, extends, and recognizes the linear pattern by its rule, such as the total number of legs on a given number of horses can be calculated by counting by fours.

A B The student explained what patterns are, and noted that a pattern can be linear, i.e., that it "may or may not repeat itself." The student gave an example of a linear pattern and extended it.

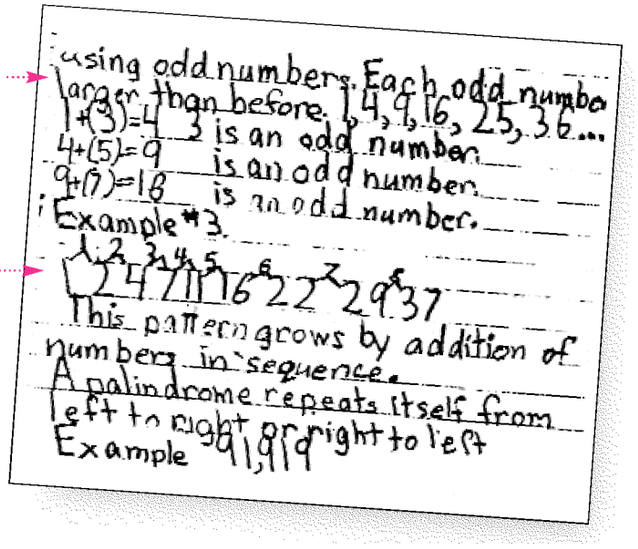
M3 b Function and Algebra Concepts: The student builds iterations of simple non-linear patterns, including multiplicative and squaring patterns (e.g., "growing" patterns) with concrete materials, and recognizes that these patterns are not linear.

A The student explained what patterns are, and noted that a pattern can "grow or lower itself," i.e., be non-linear.

C D The student went on to give two examples of different kinds of non-linear patterns: one in which the numbers get larger by adding the next odd number in sequence; the other in which the numbers get larger by adding "numbers in sequence."



① What is a mathematical pattern?
A mathematical pattern is numbers, letters or shapes or a combination of numbers, letters or shapes. The pattern may or may not repeat itself. The pattern can also grow or lower itself by addition, subtraction, multiplication or division. When you look at a mathematical pattern you must first look for a clue to figure out the secret for the rest of the pattern.
Example #1.
This is a pattern of letters and shapes that keep repeating.
abc, ΔO□, abc, ΔO□, abc, ΔO□
Example #2
This pattern grows by addition



using odd numbers. Each odd numbe larger than before. 1, 4, 9, 16, 25, 36...
1+(3)=4 3 is an odd number
4+(5)=9 is an odd number
9+(7)=16 is an odd number.
Example #3
1 2 3 4 5 6 7 8
1 4 7 11 16 22 29 37
This pattern grows by addition of numbers in sequence.
A palindrome repeats itself from left to right or right to left
Example 9, 9, 9

M7 a Mathematical Communication: The student uses appropriate mathematical terms, vocabulary, and language, based on prior conceptual work.

A B C D These parts of the work provide evidence of appropriate use of mathematical terms and vocabulary. This evidence is supported throughout the work.

The quotations from the Mathematics performance descriptions in this commentary are excerpted. The complete performance descriptions are shown on pages 60-65.

This work sample illustrates a standard-setting performance for the following parts of the standards:

M3 a Function and Algebra Concepts: Use linear patterns to solve problems.

M3 b Function and Algebra Concepts: Build iterations of simple non-linear patterns.

M7 a Mathematical Communication: Use appropriate mathematical terms, vocabulary, and language.

Work Sample & Commentary: *Pumpkin Activity*

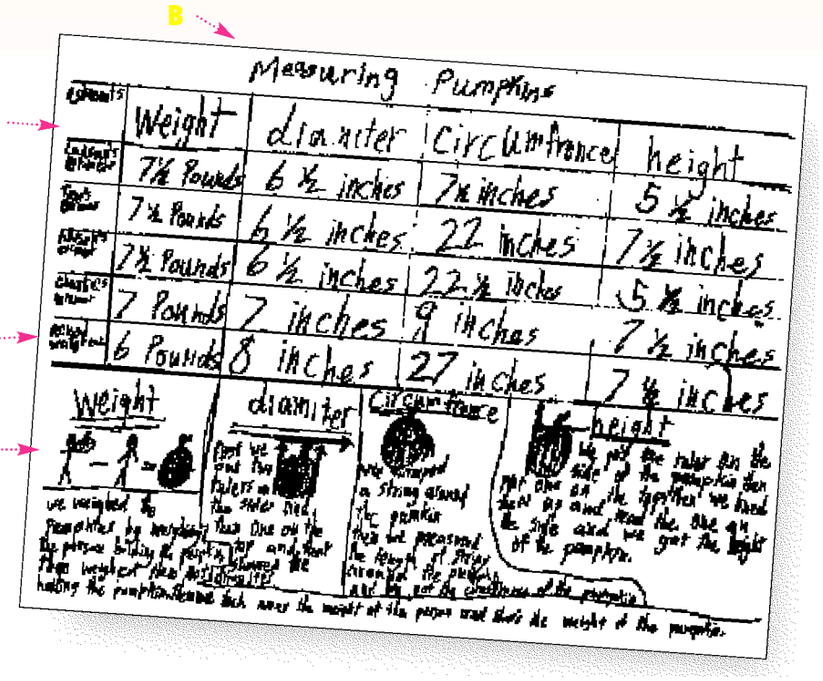
Arithmetic and Number Concepts
Geometry & Measurement Concepts M2
Function & Algebra Concepts
Statistics & Probability Concepts M4
Problem Solving & Reasoning
Mathematical Skills & Tools M6
Mathematical Communication M7
Putting Mathematics to Work

The quotations from the Mathematics performance descriptions in this commentary are excerpted. The complete performance descriptions are shown on pages 60-65.

The task

The teacher gave these instructions:

- Estimate and then measure the height, diameter, circumference, and weight of your group's pumpkin. Use the pieces of yarn, rulers, and a bathroom scale.
- Record your estimated and actual measurements in a clear and organized way.
- Explain how you made the measurements of each feature you are measuring.
- Estimate the number of seeds in your pumpkin, decide on a counting method, and then count the seeds.
- Record what true statements or conclusions you can make about the data from all eight class pumpkins.



Circumstances of performance

This sample of student work was produced under the following conditions:

alone	in a group
in class	as homework
with teacher feedback	with peer feedback
timed	opportunity for revision

While students shared pumpkins in groups of four, they made their estimates and recorded the data individually. The teacher verified that students measured correctly.

What the work shows

M2 g Geometry and Measurement Concepts: The student uses basic ways of estimating and measuring the size of figures and objects in the real world including length, width, perimeter, and area.

A The student estimated, measured, recorded, and described the weight, diameter, circumference, and height of a pumpkin.

M4 b Statistics and Probability Concepts: The student displays data in…tables and charts.

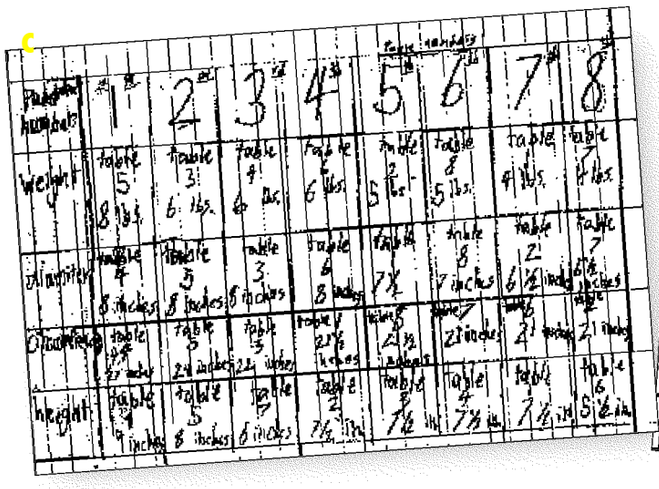
B C The student produced a chart and a table that provide clear displays of data about the group's estimates and measurements and about the data from the whole class.

This work sample illustrates a standard-setting performance for the following parts of the standards:

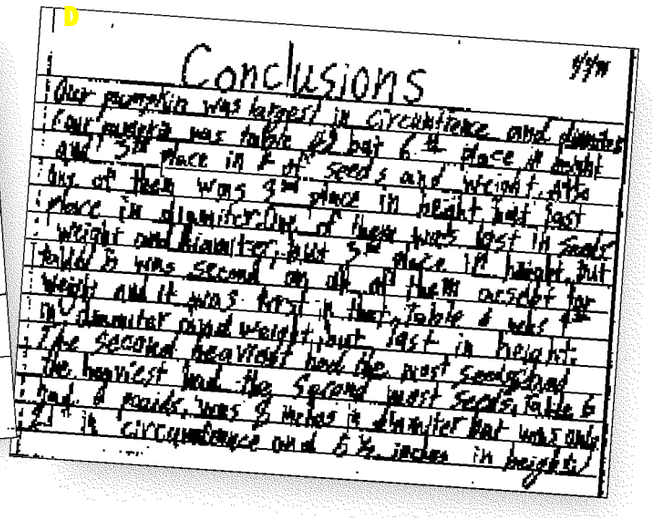
- **M2 g Geometry and Measurement Concepts: Use basic ways of estimating and measuring the size of figures and objects in the real world.**
- **M4 b Statistics and Probability Concepts: Display data.**
- **M4 c Statistics and Probability Concepts: Make statements and draw simple conclusions based on data.**
- **M6 b Mathematical Skills and Tools: Estimate numerically and spatially.**
- **M6 c Mathematical Skills and Tools: Measure accurately.**
- **M6 g Mathematical Skills and Tools: Read, create, and represent data.**
- **M6 h Mathematical Skills and Tools: Use measuring devices.**
- **M7 a Mathematical Communication: Use appropriate mathematical terms, vocabulary, and language.**
- **M7 b Mathematical Communication: Show mathematical ideas in a variety of ways.**

Pumpkin Activity

C



D



M4 c Statistics and Probability Concepts: The student makes statements…based on data; that is, reads data in…tables and charts; compares data in order to make true statements….

D The student read the data about the class pumpkins in order to make comparisons and produce true statements about them.

M6 b Mathematical Skills and Tools: The student estimates numerically and spatially.

E The student estimated weight, diameter, circumference, and height.

M6 c Mathematical Skills and Tools: The student measures length,…circumference, diameter,…weight accurately….

F The student measured weight, diameter, circumference, and height accurately. These measurements were verified by the teacher during the activity.

M6 g Mathematical Skills and Tools: The student reads, creates, and represents data on…charts, tables….

B C The chart and table are correctly drawn.

M6 h Mathematical Skills and Tools: The student uses…measuring devices…to achieve solutions.

A F The student used rulers and yarn to find measurements.

M7 a Mathematical Communication: The student uses appropriate mathematical terms, vocabulary, and language, based on prior conceptual work.

D True statements based on the data are communicated clearly.

M7 b Mathematical Communication: The student shows mathematical ideas in a variety of ways, including words, numbers, symbols, pictures, charts, graphs, tables, diagrams, and models.

A B C The process of measurement is explained in multiple ways (words, diagrams, symbolic notation) and the data are represented in a clear and easily interpreted chart and table.

The work includes some misspelled words. This sample represents work done in class which was not further edited for spelling. The errors occur in words that represented relatively new concepts for the student, e.g., "estiment" for "estimate" and "circumfrence" for "circumference."

The following table shows the number of seeds in each pumpkin reported by table groups. The student referred to these data in the "Conclusions" section of the response.

	Table 1	Table 2	Table 3	Table 4	Table 5	Table 6	Table 7	Table 8
Number of seeds	369	600	824	738	793	562	233	486

Work Sample & Commentary: *Height Measurement Statistics*

Arithmetic and Number Concepts
Geometry & Measurement Concepts
Function & Algebra Concepts
Statistics & Probability Concepts M4
Problem Solving & Reasoning
Mathematical Skills & Tools
Mathematical Communication M7
Putting Mathematics to Work

The quotations from the Mathematics performance descriptions in this commentary are excerpted. The complete performance descriptions are shown on pages 60-65.

The task

The teacher gave the following instructions:

- Collect everyone's height measurement in inches.
- Make a line plot with the data.
- Write about what you noticed about the data.

Circumstances of performance

This sample of student work was produced under the following conditions:

alone	in a group
in class	as homework
with teacher feedback	with peer feedback
timed	opportunity for revision

What the work shows

M4 a Statistics and Probability Concepts: The student collects and organizes data to answer a question….

A **B** The student began with the question of how tall her classmates were, and collected and organized data to answer this question.

M4 b Statistics and Probability Concepts: The student displays data in line plots….

B

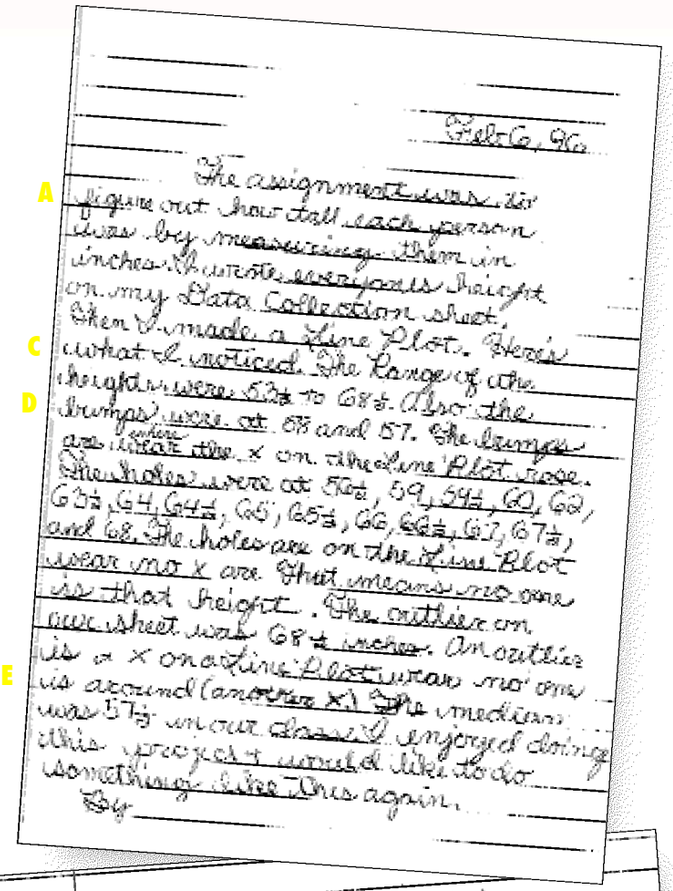
M4 c Statistics and Probability Concepts: The student makes statements and draws simple conclusions based on data; that is, the student:

- **reads data in line plots….**

B

- **compares data in order to make true statements, e.g., "seven plants grew at least 7 cm."**

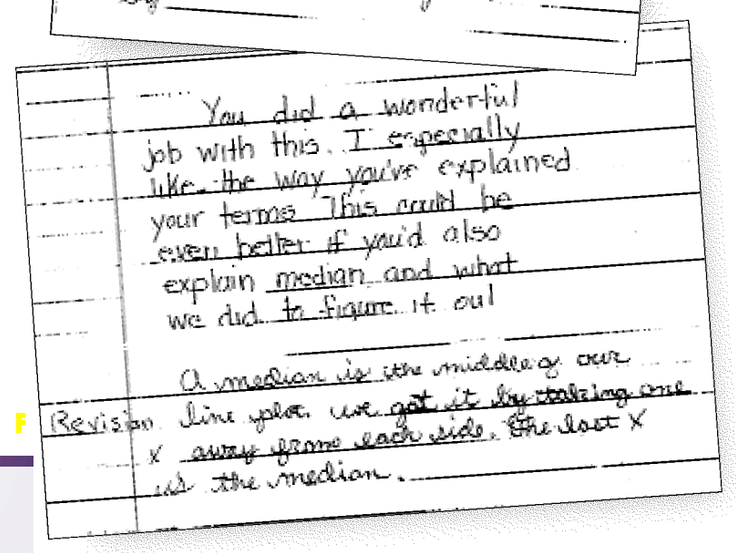
C The student made true statements about the range, "bumps," "holes," outlier, and median.



Feb 6, 96

The assignment was to figure out how tall each person was by measuring them in inches. I wrote everyone's height on my Data Collection sheet. Then I made a Line Plot. Here's what I noticed. The Range of the heights were 53½ to 68½. Also the bumps were at 58 and 57. The bumps are where the x on the Line Plot are. The holes were at 56½, 59, 59½, 60, 62, 63½, 64, 64½, 65, 65½, 66, 66½, 67, 67½, and 68. The holes are on the Line Plot where no x are. That means no one is that height. The outlier on our sheet was 68½ inches. An outlier is a x on a Line Plot where no one is around (another x). The median was 57½ in our class. I enjoyed doing this project. I would like to do something like this again.

By



You did a wonderful job with this. I especially like the way you've explained your terms. This could be even better if you'd also explain median and what we did to figure it out

A median is the middle of our line plot. We got it by taking one x away from each side. The last x is the median.

This work sample illustrates a standard-setting performance for the following parts of the standards:

M4 a Statistics and Probability Concepts: Collect and organize data to answer a question.

M4 b Statistics and Probability Concepts: Display data.

M4 c Statistics and Probability Concepts: Make statements and draw simple conclusions based on data.

M7 a Mathematical Communication: Use appropriate mathematical terms, vocabulary, and language.

- **identifies and uses the mode necessary for making true statements, e.g., "more people chose red."**

D

- **makes true statements based on a simple concept of average (median and mean), for a small sample size and where the situation is made evident with concrete materials or clear representations.**

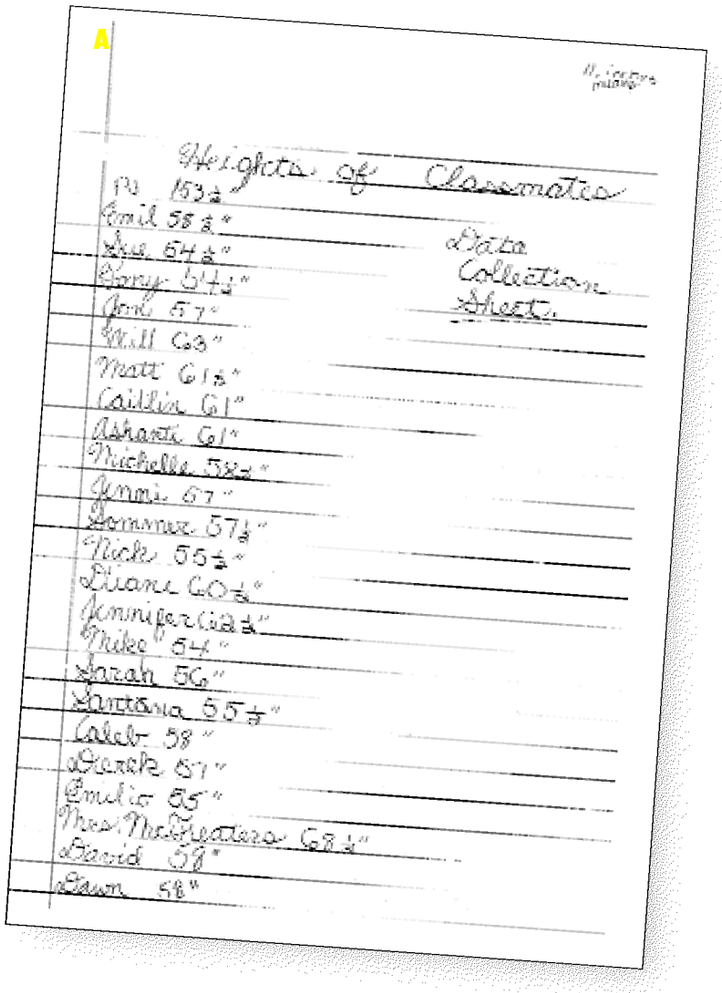
E

F The revision provides further evidence of the student's understanding.

Height Measurement Statistics

M7 a Mathematical Communication: The student uses appropriate mathematical terms, vocabulary, and language, based on prior conceptual work.

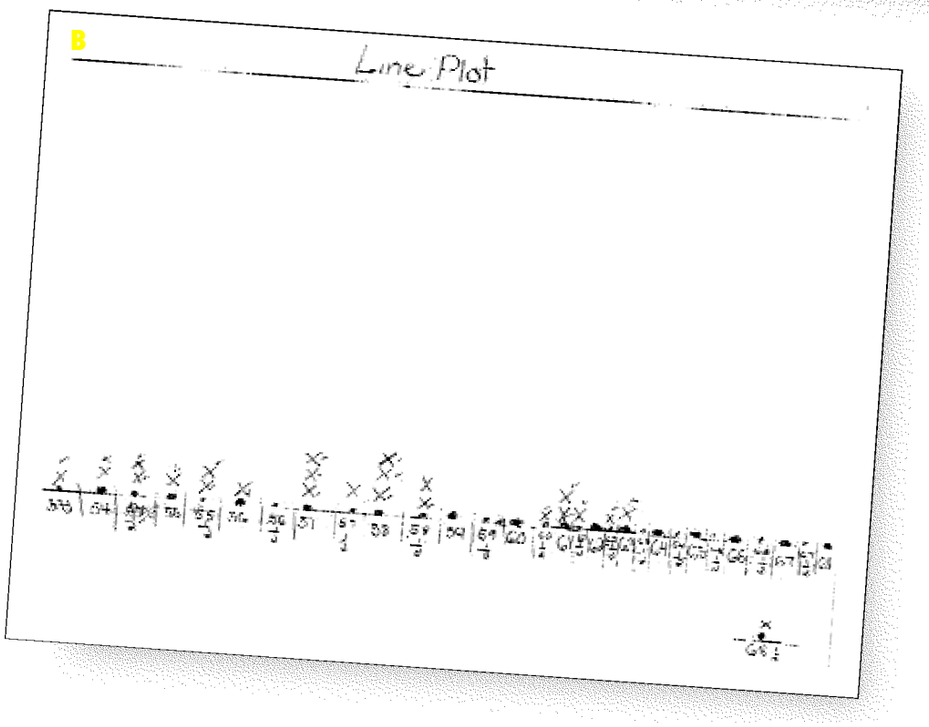
A C D E Appropriate use of mathematical terms and vocabulary is evident in these parts of the work particularly, and is supported by evidence throughout the work.


A

Heights of Classmates

Data Collection Sheet

Me 153½
Emil 58½"
Sue 64½"
Gary 64½"
Jack 57"
Will 63"
Matt 61½"
Caitlin 61"
Ashanti 61"
Michelle 58½"
Jenni 57"
Jonnnie 57½"
Nick 55½"
Liane 60½"
Jennifer 61½"
Mike 54"
Sarah 56"
Santana 55½"
Caleb 58"
Derek 51"
Emilio 65"
Mrs. McCreadera 68½"
David 58"
Dawn 58"


B

Line Plot

Work Sample & Commentary: *Two Dice Sums*

Arithmetic and Number Concepts M1
Geometry & Measurement Concepts
Function & Algebra Concepts
Statistics & Probability Concepts M4
Problem Solving & Reasoning
Mathematical Skills & Tools M6
Mathematical Communication M7
Putting Mathematics to Work

The quotations from the Mathematics performance descriptions in this commentary are excerpted. The complete performance descriptions are shown on pages 60-65.

The task

After students had many chances to roll two dice and record the sums, the teacher gave the following instructions for this multi-part probability activity:

1. Think of a way to figure out all the combinations (sums) for two dice rolled together. Write a title and explain what you did. We will discuss the various strategies you came up with, and look at what works well.

2. Use a 6 x 6 grid to figure out the combinations for two dice sums to check that you found all the ways. (The teacher provided an example of a 6 x 6 grid, without supplying the numbers in the inner cells.)

When you are finished, show how each sum can be shown as a fraction of the total number of combinations that are possible.

3. Play this game with a partner several times:

- Draw a chart with spaces under the sums for two dice.
- Draw 11 circles in any of the 11 spaces in any combination you want.
- Write the reason why you put the circles where you did. Try to use fractions to explain your choices.
- With a partner, take turns rolling the dice. Each time you roll a sum, put Xs in all the circles you have for that sum. You win when all your circles are crossed off.

This game is based on an activity from *A Collection of Math Lessons, 3 - 6,* Marilyn Burns, Math Solution Publications, 1987.

This work sample illustrates a standard-setting performance for the following parts of the standards:

M1 d Arithmetic and Number Concepts: Describe and compare quantities by using simple fractions.

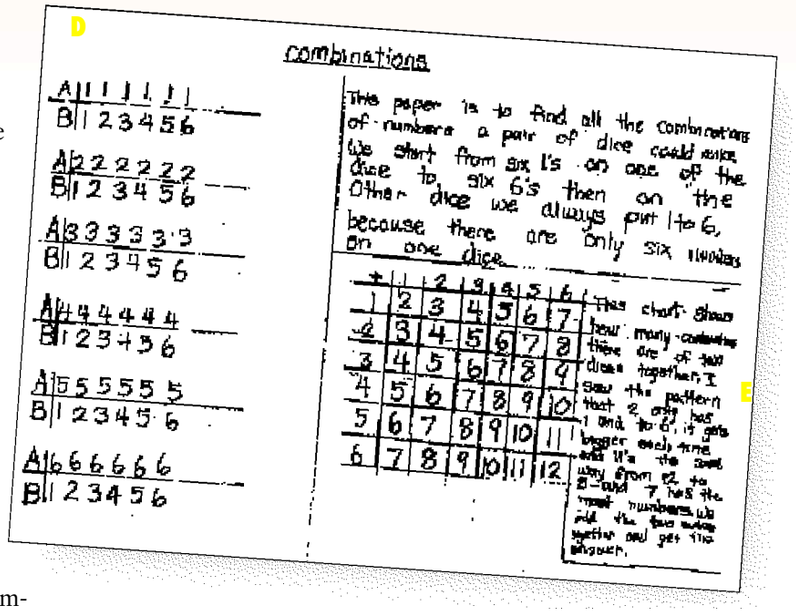
M4 e Statistics and Probability Concepts: Predict results and find out why some results are more likely, less likely, or equally likely.

M4 f Statistics and Probability Concepts: Find all possible combinations.

M6 g Mathematical Skills and Tools: Read, create, and represent data.

M7 b Mathematical Communication: Show mathematical ideas in a variety of ways.

M7 c Mathematical Communication: Explain solutions to problems clearly and logically.



Circumstances of performance

This sample of student work was produced under the following conditions:

alone	in a group
in class	as homework
with teacher feedback	with peer feedback
timed	opportunity for revision

Students had prior experience of making their own charts and using conventional fractional notation, and had explored probability using dice and other materials. The activities were spread out over two days.

What the work shows

M1 d Arithmetic and Number Concepts: The student describes and compares quantities by using concrete and real world models of simple fractions; that is, finds simple parts of wholes….

A All possible sums for paired dice are represented and compared as simple fractions.

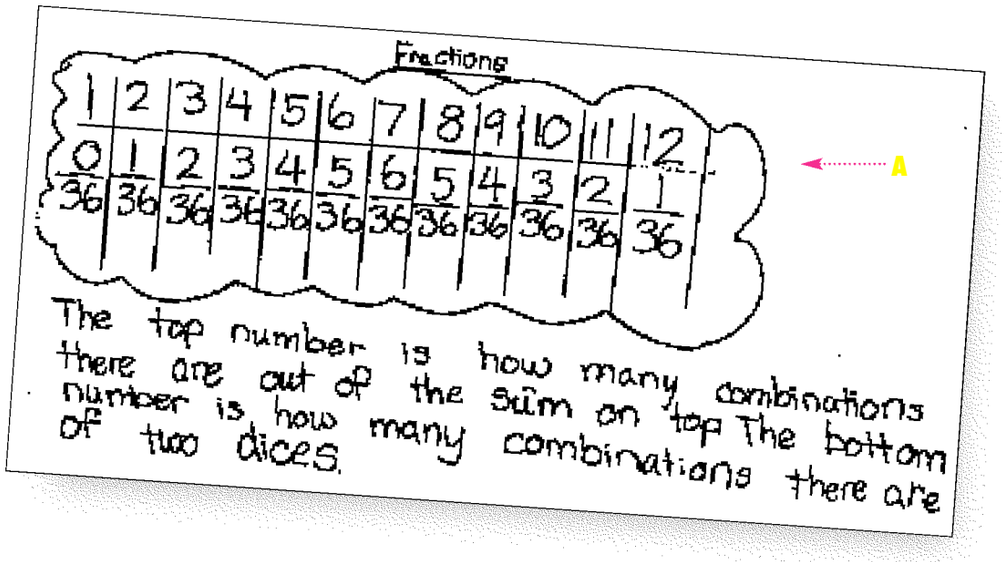
M4 e Statistics and Probability Concepts: The student predicts results, analyzes data, and finds out why some results are more likely, less likely, or equally likely.

A B C The student organized the possible outcomes in order to make predictions about the likelihood of rolling particular two dice sums.

M4 f Statistics and Probability Concepts: The student finds all possible combinations and arrangements within certain constraints involving a limited number of variables.

D The student created a way to find all possible combinations for the sums of two dice.

Two Dice Sums



Fractions

1	2	3	4	5	6	7	8	9	10	11	12
0/36	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

The top number is how many combinations there are out of the sum on top The bottom number is how many combinations there are of two dices.

A

M6 g Mathematical Skills and Tools: The student reads, creates, and represents data on…charts, tables….

A D The student-created charts, especially the horizontal tables under "Combinations" and the fractions chart, are appropriate, clear, and complete.

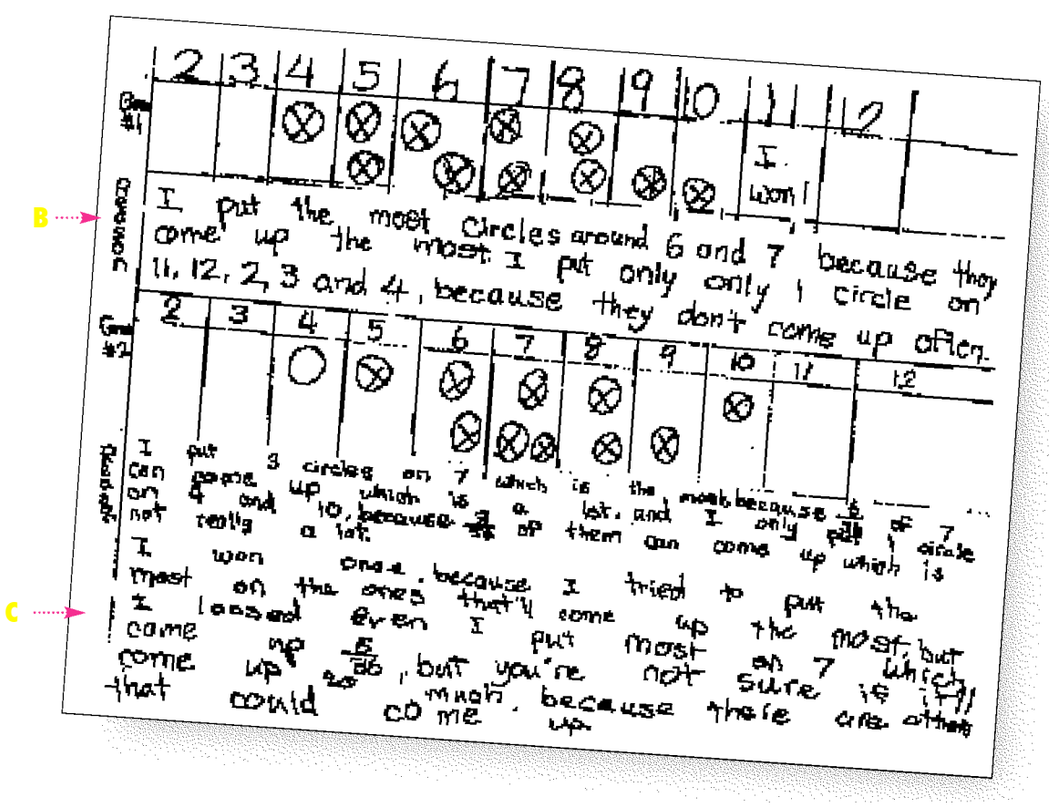
M7 b Mathematical Communication: The student shows mathematical ideas in a variety of ways, including words, numbers, symbols,…charts,… tables….

D E The student used words to explain how the chart works. Numbers, symbols ("A" and "B"), and a table are used to communicate the combinations.

M7 c Mathematical Communication: The student explains solutions to problems clearly and logically, and supports solutions with evidence, in… written work.

B C The written explanations are sufficiently clear and logical for the elementary level. The student provided evidence in support of the game strategy by referring back to the data contained in the tables.

The work includes several errors in usage and grammar, e.g., "two dices," "loosed" for "lost," and "come" for "came." This was a class assignment and was not intended to be further edited.



Game #1

2	3	4	5	6	7	8	9	10	11	12
		⊗	⊗	⊗	⊗	⊗				
			⊗	⊗	⊗	⊗	⊗	⊗	I won!	

B

I put the most circles around 6 and 7 because they come up the most I put only only 1 circle on 11, 12, 2, 3 and 4, because they don't come up often.

Game #2

2	3	4	5	6	7	8	9	10	11	12
		○	⊗	⊗	⊗	⊗				
				⊗	⊗	⊗	⊗	⊗		

I put 3 circles on 7 which is the most because 5/36 of 7 can come up which is a lot, and I only put 1 circle on 4 and 10, because 3/36 of them can come up which is not really a lot.

I won once, because I tried to put the most on the ones that'll come up the most, but I loosed even I put most on 7 which come up 6/36, but you're not sure if it could come up much, because there are others that could come up.

C

Work Sample & Commentary: *Creatures*

Arithmetic and Number Concepts
Geometry & Measurement Concepts
Function & Algebra Concepts
Statistics & Probability Concepts M4
Problem Solving & Reasoning M5
Mathematical Skills & Tools
Mathematical Communication M7
Putting Mathematics to Work

The quotations from the Mathematics performance descriptions in this commentary are excerpted. The complete performance descriptions are shown on pages 60-65.

The task

The teacher asked the class to draw four Halloween characters (a ghost, a witch, a skeleton, and a pumpkin-headed scarecrow). She then instructed them to cut the figures into four heads and four bodies and to staple each set into a small "flip book."

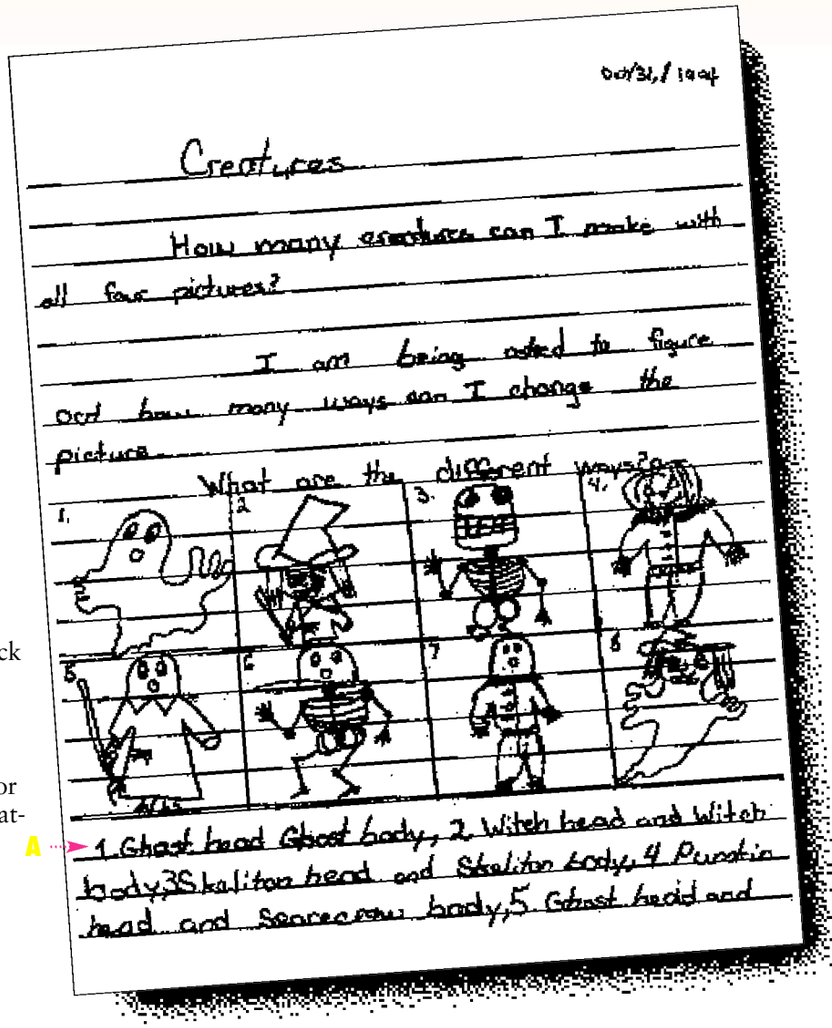
The teacher asked: How many characters could you possibly come up with by combining the different parts in various ways? Show and explain in detail all the combinations you could make.

Circumstances of performance

This sample of student work was produced under the following conditions:

alone	in a group
in class	as homework
with teacher feedback	with peer feedback
timed	opportunity for revision

The task was assigned as a two-day homework "problem of the week." There had been no prior work by the class with similar discrete mathematics or logic tasks.



A

Mathematics required by the task

In this task the teacher instructed students to make and use a "flip book" as a tool to help find all possible combinations. So, while the task calls for extensive problem solving and reasoning, important decisions about what materials and approach to use in formulating the problem were made for the students.

For this reason the task does not provide an opportunity for students to demonstrate the formulation part (M5 a) of the Problem Solving and Reasoning standard. The task does, however, provide an opportunity for students to demonstrate the implementation (M5 b) and conclusion (M5 c) parts of the standard.

What the work shows

M4 f Statistics and Probability Concepts: The student finds all possible combinations and arrangements within certain constraints involving a limited number of variables.

A The student found all the possible combinations of the four heads and four bodies.

This work sample illustrates a standard-setting performance for the following parts of the standards:

- M4 f **Statistics and Probability Concepts: Find all possible combinations.**
- M5 b **Problem Solving and Reasoning: Implementation.**
- M5 c **Problem Solving and Reasoning: Conclusion.**
- M7 b **Mathematical Communication: Show mathematical ideas in a variety of ways.**
- M7 c **Mathematical Communication: Explain solutions to problems clearly and logically.**

Creatures

M5 b Problem Solving and Reasoning: The student makes the basic choices involved in planning and carrying out a solution; that is,…

- **makes up and uses a variety of strategies and approaches to solving problems….**

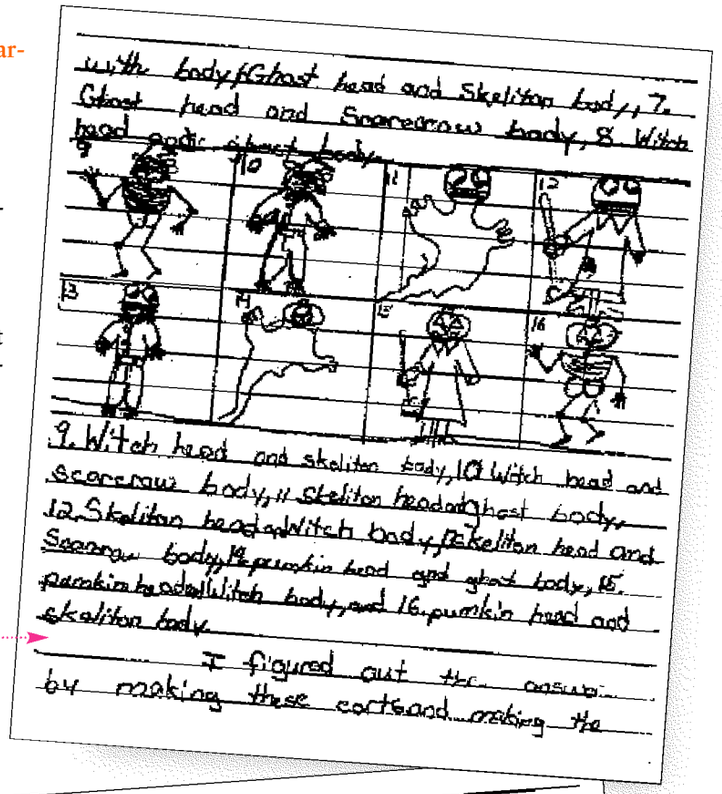
B The student used two approaches to solve the problem: a "flip book"/systematic listing and a student-created multiplicative formula.

- **makes connections among concepts in order to solve problems….**

C The student made connections among different conceptual approaches: "I know that my [systematic listing] solution is correct because there are four monsters, so that means there are four bodies [and four heads] and four times four equals sixteen."

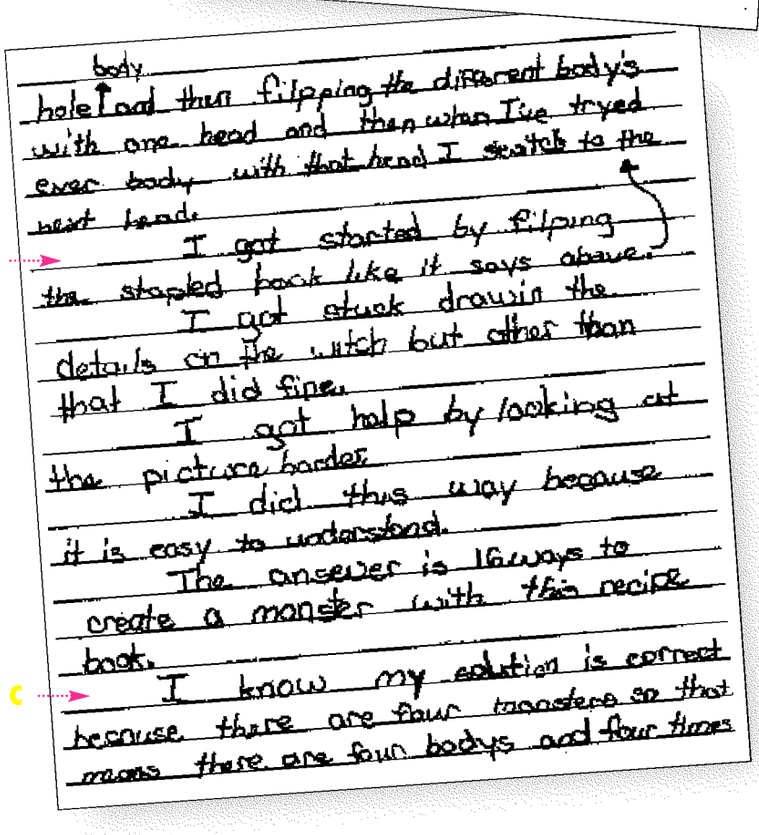
- **solves problems in ways that make sense and explains why these ways make sense….**

B C D The student explained why the solution made sense in several ways by setting out clearly both of the approaches used.


with body, 6 Ghost head and Skeliton body, 7. Ghost head and Scarecrow body, 8 Witch head and ghost body.

9. Witch head and skeliton body, 10 Witch head and Scarecrow body, 11 Skeliton head and ghost body, 12. Skeliton head and Witch body, 13 Skeliton head and Scarecrow body, 14 pumkin head and ghost body, 15. pumkin head and Witch body, and 16. pumkin head and skeliton body.

B I figured out the answer by making these cards and making the


body hole and then filpping the different body's with one head and then when I've tryed every body with that head I switch to the next head.

D I got started by filping the stapled book like it says above.

I got stuck drawin the details on the witch but other than that I did fine.

I got help by looking at the picture border.

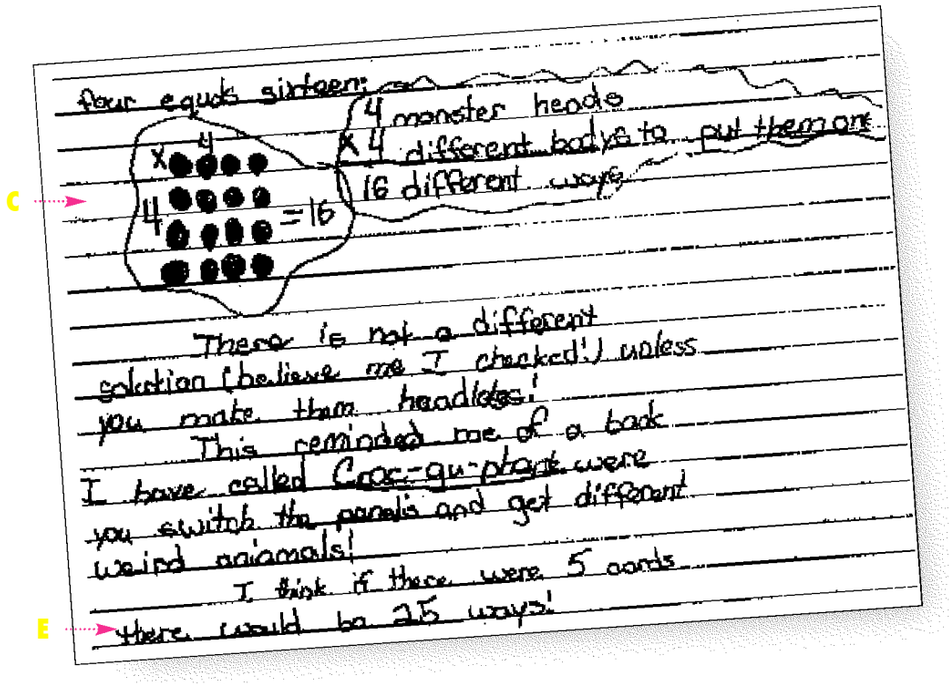
I did this way because it is easy to understand.

The answer is 16 ways to create a monster with this recipe book.

C I know my solution is correct because there are four monsters so that means there are four bodys and four times

Creatures

Arithmetic and Number Concepts
Geometry& Measurement Concepts
Function & Algebra Concepts
Statistics & Probability Concepts M4
Problem Solving & Reasoning M5
Mathematical Skills & Tools
Mathematical Communication M7
Putting Mathematics to Work



M5 c Problem Solving and Reasoning: The student moves beyond the particular problem by making…extensions, and/or generalizations; for example,…makes the solution into a general rule that applies to other circumstances.

E The student moved beyond the particular problem by defining a general rule (four heads times four bodies) and applying it in a different circumstance: "I think if there were five cards there would be 25 ways."

M7 b Mathematical Communication: The student shows mathematical ideas in a variety of ways, including words, numbers, symbols, pictures….

A C The systematic listing makes use of both pictures and sentences. Numbers and a diagram are used to represent the multiplicative formula.

M7 c Mathematical Communication: The student explains solutions to problems clearly and logically, and supports solutions with evidence….

A C D The explanation is clear, logical, and supported with diagrammatic, numeric, pictorial, and narrative evidence.

The work includes several misspelled words in the written portion of the work (e.g., "skeliton," "pumkin," and "carts" instead of "charts"). This was a homework assignment and was not intended to be further edited for spelling.

Work Sample & Commentary: *¿Se hace un triangulo?*

The task

The teacher gave the students the following instructions:

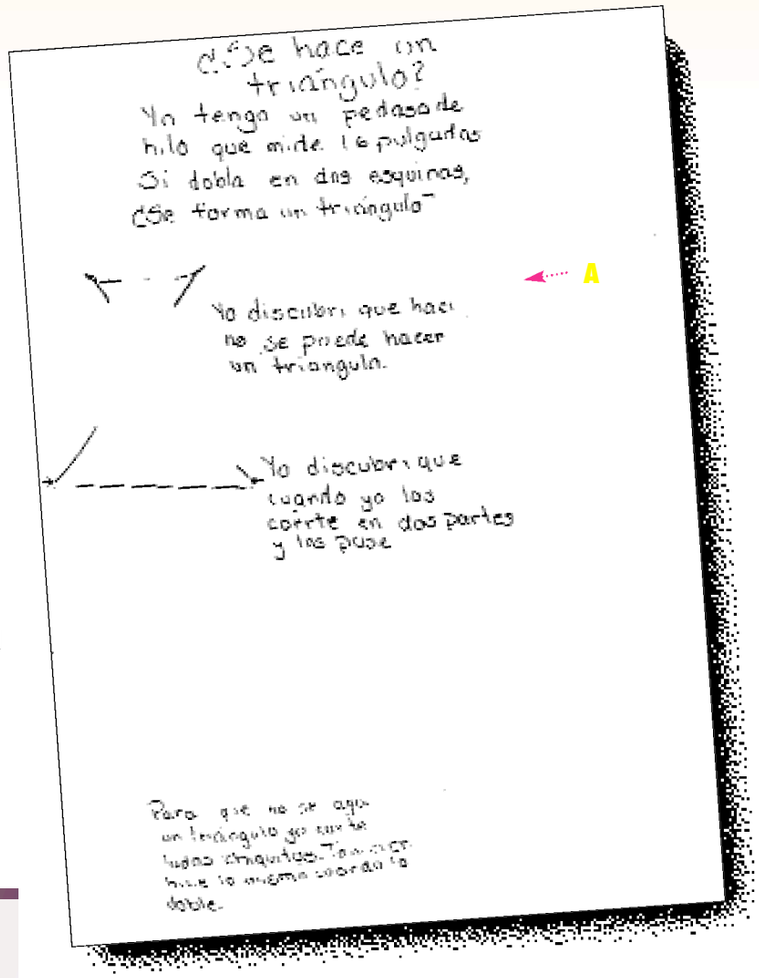
Suppose you were given a string that is sixteen inches long. If you cut or fold it in any two places, will it always make a triangle?

Circumstances of performance

This sample of student work was produced under the following conditions:

alone	in a group
in class	as homework
with teacher feedback	with peer feedback
timed	opportunity for revision

This sample was completed in Spanish in a bilingual classroom. The translation was provided by the teacher.


¿Se hace un triangulo?
Yo tengo un pedazo de hilo que mide 16 pulgadas Si dobla en dos esquinas, ¿Se forma un triangulo?

A

Yo descubri que hace no se puede hacer un triangulo.

Yo descubri que cuando yo los corte en dos partes y los pose

Para que se haga un triangulo yo creo que hacer dos triangulos. Tienen que hacer lo mismo cuando lo doble.

This work sample illustrates a standard-setting performance for the following parts of the standards:

- **M2 d Geometry and Measurement Concepts: Use many types of figures.**
- **M2 e Geometry and Measurement Concepts: Solve problems by showing relationships between and among figures.**
- **M2 g Geometry and Measurement Concepts: Use basic ways of estimating and measuring the size of figures and objects in the real world.**
- **M2 i Geometry and Measurement Concepts: Select and use units for estimating and measuring length.**
- **M5 a Problem Solving and Reasoning: Formulation.**
- **M5 b Problem Solving and Reasoning: Implementation.**
- **M5 c Problem Solving and Reasoning: Conclusion.**
- **M6 e Mathematical Skills and Tools: Refer to geometric shapes and terms correctly.**
- **M7 b Mathematical Communication: Show mathematical ideas in a variety of ways.**
- **M7 c Mathematical Communication: Explain solutions to problems clearly and logically.**

What the work shows

M2 d Geometry and Measurement Concepts: The student uses many types of figures (angles, triangles…) and identifies the figures by their properties….

A B C The student used angles and triangles throughout this problem solving activity.

D The student did not merely identify, but discovered a property for, triangles: "if the two sides add up to more than the bottom side, it will make a triangle."

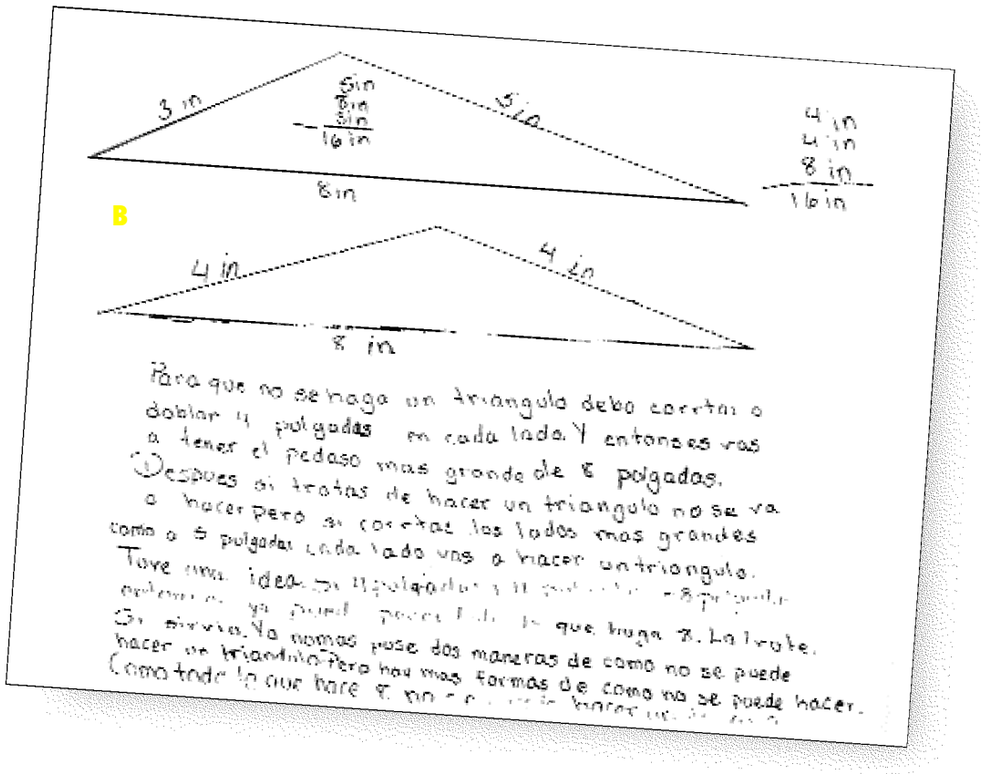
M2 e Geometry and Measurement Concepts: The student solves problems by showing relationships between and among figures….

A B C The student searched for and showed relationships among sides of triangles.

D The student summarized relationships among sides of triangles in a general rule.

The quotations from the Mathematics performance descriptions in this commentary are excerpted. The complete performance descriptions are shown on pages 60-65.

¿Se hace un triangulo?



Arithmetic and Number Concepts
Geometry & Measurement Concepts M2
Function & Algebra Concepts
Statistics & Probability Concepts
Problem Solving & Reasoning M5
Mathematical Skills & Tools M6
Mathematical Communication M7
Putting Mathematics to Work

M2 g Geometry and Measurement Concepts: The student uses basic ways of estimating and measuring the size of figures and objects in the real world, including length,…perimeter….

A B C

M2 i Geometry and Measurement Concepts: The student selects and uses units, both formal and informal as appropriate, for estimating and measuring quantities such as…length….

B C

M5 a Problem Solving and Reasoning: Formulation. Given the basic statement of a problem situation, the student:

- makes the important decisions about the approach, materials, and strategies to use, i.e., does not merely fill in a given chart, use a pre-specified manipulative, or go through a predetermined set of steps.

A The student decided to use a trial-and-error approach, which later became more systematic. The student also decided to use string, scissors, and a ruler.

- uses previously learned strategies, skills, knowledge, and concepts to make decisions.

A The student used knowledge about triangles and measurement to develop the approach.

- uses strategies, such as using manipulatives or drawing sketches, to model problems.

A B

M5 b Problem Solving and Reasoning: Implementation. The student makes the basic choices involved in planning and carrying out a solution; that is, the student:

- makes up and uses a variety of strategies and approaches to solving problems and uses or learns approaches that other people use, as appropriate.

B The student used measurement strategies to solve the problem.

- makes connections among concepts in order to solve problems.

B C The student made connections between measurement and geometry concepts to solve the problem.

- solves problems in ways that make sense and explains why these ways make sense, e.g., defends the reasoning, explains the solution.

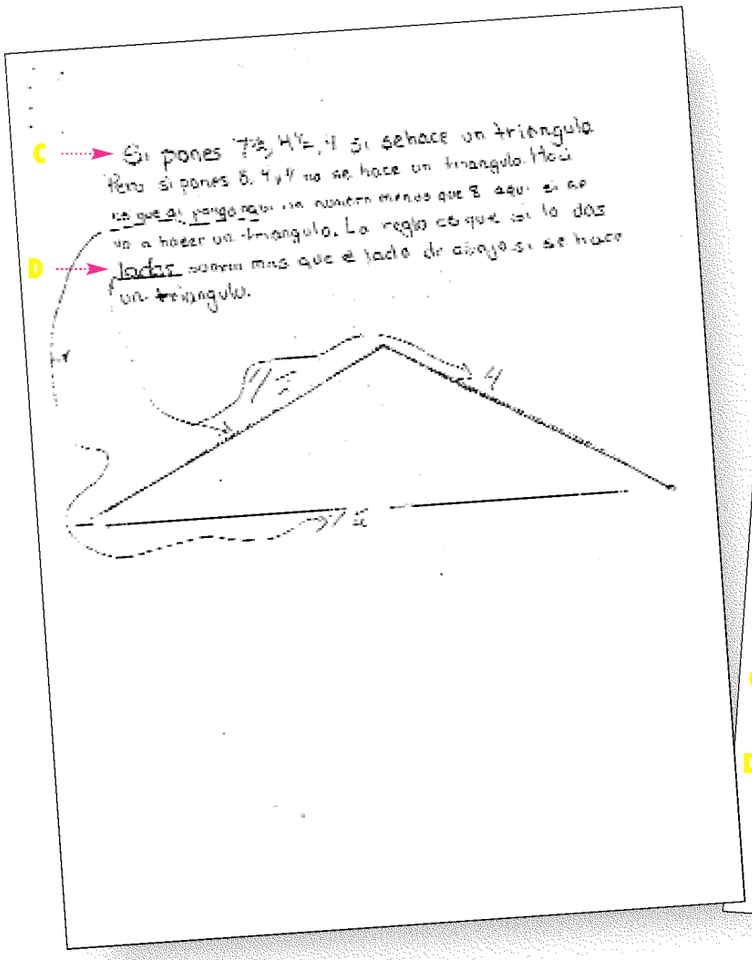
A B C

M5 c Problem Solving and Reasoning: Conclusion. The student moves beyond a particular problem by making connections, extensions, and/or generalizations; for example, the student:…

- makes the solution into a general rule that applies to other circumstances.

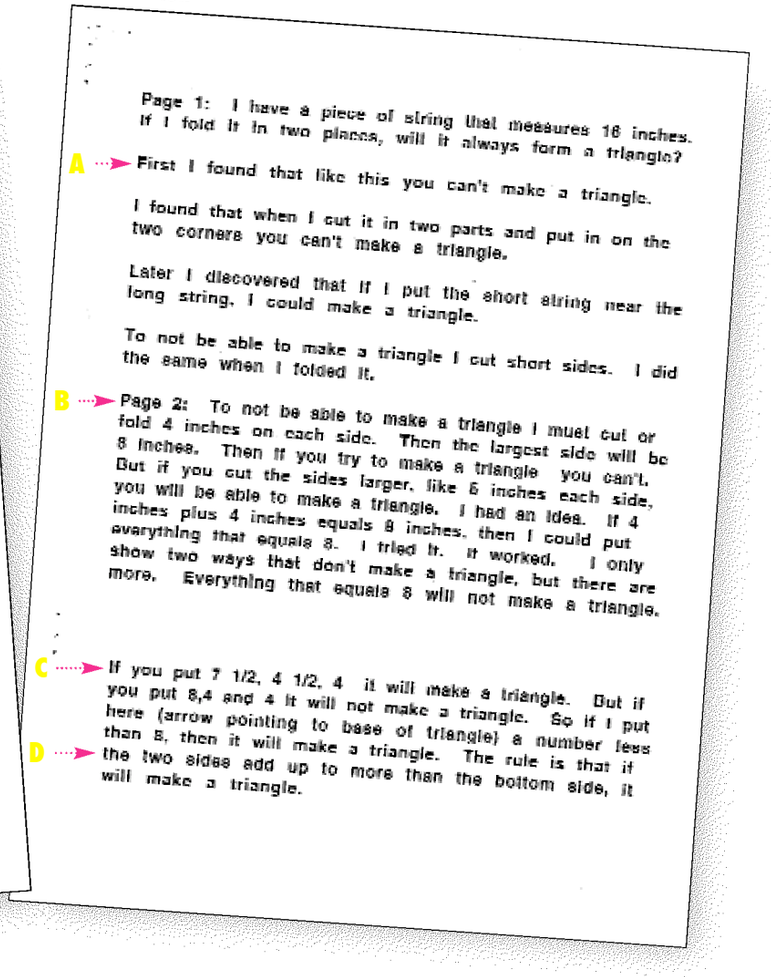
D The student went beyond the problem by finding a rule that would work for making any triangle.

¿Se hace un triangulo?



C Si pones 7½, 4½, 4 si se hace un triangulo
Pero si pones 8, 4, 4 no se hace un triangulo Hoy
se que si pongo un numero menos que 8 aqui se va
a hacer un triangulo. La regla es que si lo dos
D lados suman mas que el lado de abajo si se hace
un triangulo.

Translation



Page 1: I have a piece of string that measures 16 inches. If I fold it in two places, will it always form a triangle?

A First I found that like this you can't make a triangle.

I found that when I cut it in two parts and put in on the two corners you can't make a triangle.

Later I discovered that if I put the short string near the long string, I could make a triangle.

To not be able to make a triangle I cut short sides. I did the same when I folded it.

B Page 2: To not be able to make a triangle I must cut or fold 4 inches on each side. Then the largest side will be 8 inches. Then if you try to make a triangle you can't. But if you cut the sides larger, like 5 inches each side, you will be able to make a triangle. I had an idea. If 4 inches plus 4 inches equals 8 inches, then I could put everything that equals 8. I tried it. it worked. I only show two ways that don't make a triangle, but there are more. Everything that equals 8 will not make a triangle.

C If you put 7 1/2, 4 1/2, 4 it will make a triangle. But if you put 8,4 and 4 it will not make a triangle. So if I put here (arrow pointing to base of triangle) a number less than 8, then it will make a triangle. The rule is that if
D the two sides add up to more than the bottom side, it will make a triangle.

M6 e Mathematical Skills and Tools: The student refers to geometric shapes and terms correctly with concrete objects or drawings, including triangle,…side….

A B C D

M7 b Mathematical Communication: The student shows mathematical ideas in a variety of ways, including words, numbers,…pictures,…diagrams….

A B C D

M7 c Mathematical Communication: The student explains solutions to problems clearly and logically, and supports solutions with evidence, in both written and oral work.

A B C D

Work Sample & Commentary: *The Great Fish Dilemma*

Arithmetic and Number Concepts
Geometry & Measurement Concepts
Function & Algebra Concepts
Statistics & Probability Concepts
Problem Solving & Reasoning M5
Mathematical Skills & Tools
Mathematical Communication
Putting Mathematics to Work

The quotations from the Mathematics performance descriptions in this commentary are excerpted. The complete performance descriptions are shown on pages 60-65.

The task

The teacher gave students the following written prompt:

How many different ways can you put nine fish in two bowls?

Show all your work and at the end explain why you made the decisions you did as you solved the problem.

Circumstances of performance

These samples of student work were produced under the following conditions:

alone	in a group
in class	as homework
with teacher feedback	with peer feedback
timed	opportunity for revision

Students could choose to include their work in the Vermont statewide portfolio assessment. They had been instructed in the specific criteria that the Vermont system uses in scoring problem solving work, so they knew what was expected. (See Marge Petit and Beth Hulbert, *Learning How to Show Your Best,* Exemplars, RR1, Box 7390, Underhill, VT 05489.)

What the work shows

M5 a Problem Solving and Reasoning: Formulation. Given the basic statement of a problem situation, the student:

- makes the important decisions about the approach, materials, and strategies to use, i.e., does not merely fill in a given chart, use a pre-specified manipulative, or go through a predetermined set of steps.

A B C In Sample 1, the student made a chart. In Sample 2, the student used cubes and blocks, and then drew pictures.

- uses previously learned strategies, skills, knowledge, and concepts to make decisions.

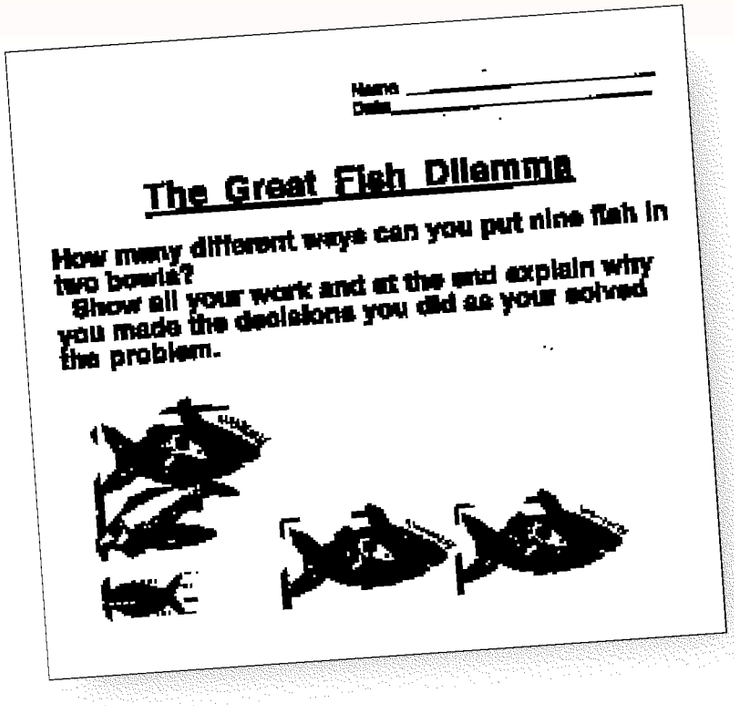
A B In Sample 1, the student used an organized chart as a strategy. In Sample 2, the student used cubes and blocks to make decisions.

These work samples illustrate standard-setting performances for the following parts of the standards:

M5 a Problem Solving and Reasoning: Formulation.

M5 b Problem Solving and Reasoning: Implementation.

M5 c Problem Solving and Reasoning: Conclusion.


Name
Date

The Great Fish Dilemma

How many different ways can you put nine fish in two bowls?
Show all your work and at the end explain why you made the decisions you did as your solved the problem.

Used by permission. Exemplars: A Teachers Solution, RR 1 Box 7390, Underhill, VT 05489.

- uses strategies, such as using manipulatives or drawing sketches, to model problems.

A B C

M5 b Problem Solving and Reasoning: Implementation. The student makes the basic choices involved in planning and carrying out a solution; that is, the student:

- makes up and uses a variety of strategies and approaches to solving problems and uses or learns approaches that other people use, as appropriate.

A B C D E In Sample 1, the student analyzed the chart to refine the solution. In Sample 2, the student used cubes, blocks, pictures, patterns, and a concept of "dividing" to solve the problem.

- makes connections among concepts in order to solve problems.

A C D Both students made connections between number concepts and the probability concept of finding all possible combinations. In Sample 2, the student also connected the concept of patterns to the concept of finding all possible combinations.

- solves problems in ways that make sense and explains why these ways make sense, e.g., defends the reasoning, explains the solution.

E F In Sample 1, the student explained why all of the possibilities "aren't good." In Sample 2, the student explained why the problem really only involved "dividing 7 fish."

The Great Fish Dilemma

Sample 1

There are nine fish and I think they are all the same. They are goldfish just like the one at my house. I made a chart to solve the problem.

A

Number of fish	Bowl 1	Bowl 2
	0	9
	1	8
	2	7
	3	6
	4	5
	5	4
	6	3
	7	2
	8	1
	9	0

Even though I found 10 possibilitys. Some aren't good. The only reason to put all fish in one bowl is if cleaning other bowl. The best choises are the ones I circled so they have room to move.

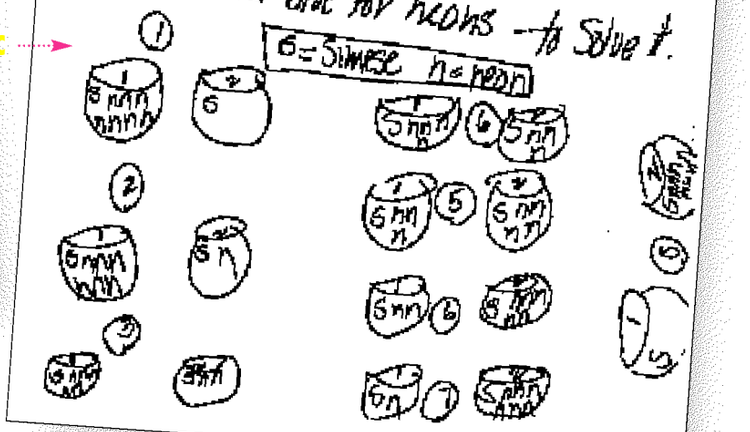
F

Sample 2

We had 9 fish and 2 bowls. Som of the fish are Siamese. Two of the fish are Siamese fighting fish, and the rest are neons. The Siamese fish can't be together ever or they kill each other. The neons can be with anytime. I drew pictures to show my answer. First I used cups and blocks—red for Siamese fish and blue for neons—to solve it.

B

C



These are all the ways because I foloed a pattern. The pattern was to put 1 Siamese fish in each bowl then start in bowl 1 with all the neons and the next time just have 6 neons in bowl 1 and 1 in bowl 2. I always had to put a Siamese fish in each bowl so I was really only divideing 7 fish (the neons). There are 8 different ways for my problem. Som are to crowded like number 1 and number 8. Neons have lots of babies then well realy be in truble.

D

E

G

M5 Problem Solving and Reasoning: Conclusion. The student moves beyond a particular problem by making connections, extensions, and/or generalizations; for example, the student:

- explains a pattern that can be used in similar situations. D
- explains how the problem solution can be applied to other school subjects and in real world situations.

F G In Sample 1, the student related the solution to the real world situation of cleaning the fishbowl. In Sample 2, the student related the solution to the real world situation of overcrowding the fish and what would happen when the fish had "babies."

Each sample includes some misspelled words, such as "choises" for "choices" and "read" for "red." The work was completed as a class exercise and was not edited to correct spelling mistakes.

Work Sample & Commentary: *How Many Handshakes?*

Arithmetic and Number Concepts
Geometry & Measurement Concepts
Function & Algebra Concepts M3
Statistics & Probability Concepts M4
Problem Solving & Reasoning M5
Mathematical Skills & Tools
Mathematical Communication M7
Putting Mathematics to Work

The quotations from the Mathematics performance descriptions in this commentary are excerpted. The complete performance descriptions are shown on pages 60-65.

The final paragraph of work sample 1 is also included among the work samples that illustrate the English Language Arts performance standards. See page 48.

The task

The teacher gave the students the following instructions:

Five people enter a room and introduce themselves to each other. If everyone shakes everyone else's hand just once, what is the total number of handshakes that occurred?

For Sample 1, the instructions were given in written form. For Sample 2, the instructions were given orally.

Circumstances of performance

These samples of student work were produced under the following conditions:

alone	in a group
in class	as homework
with teacher feedback	with peer feedback
timed	opportunity for revision

This problem was a non-routine problem for the students in both of the classrooms, which made this an appropriate problem solving task.

Sample 2 was completed in Spanish in a bilingual classroom. The translation was provided by the teacher.

These work samples illustrate standard-setting performances for the following parts of the standards:

- **M3 b Function and Algebra Concepts: Build iterations of simple non-linear patterns.**
- **M4 f Statistics and Probability Concepts: Find all possible combinations.**
- **M5 a Problem Solving and Reasoning: Formulation.**
- **M5 b Problem Solving and Reasoning: Implementation.**
- **M5 c Problem Solving and Reasoning: Conclusion.**
- **M7 b Mathematical Communication: Show mathematical ideas in a variety of ways.**
- **M7 c Mathematical Communication: Explain solutions to problems clearly and logically.**

What the work shows

M3 b Function and Algebra Concepts: The student builds iterations of simple non-linear patterns....

A B C D E F G In both samples, the students build the solution by building the pattern. They both extend this pattern into a general rule as well.

M4 f Statistics and Probability Concepts: The student finds all possible combinations and arrangements within certain constraints involving a limited number of variables.

A B H I The drawings throughout both samples show that the students found all the possible combinations of handshakes.

M5 a Problem Solving and Reasoning: Formulation. Given the basic statement of a problem situation, the student:

- **makes the important decisions about the approach, materials, and strategies to use, i.e., does not merely fill in a given chart, use a pre-specified manipulative, or go through a predetermined set of steps.**

A B H I Both students decided to use diagrams, lists, and equations.

- **uses previously learned strategies, skills, knowledge, and concepts to make decisions.**

A B H I Both samples demonstrate use of simple addition, as well as skill in drawing diagrams and making lists.

- **uses strategies, such as using manipulatives or drawing sketches, to model problems.**

A B H I

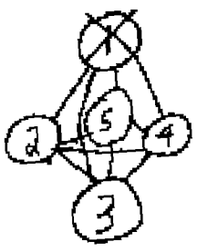
How Many Handshakes?

Sample 1

How Many Handshakes?

Five people enter a room and introduce themselves to each other. If everyone shakes everyone else's hand just once, what is the total number of handshakes that occurred.

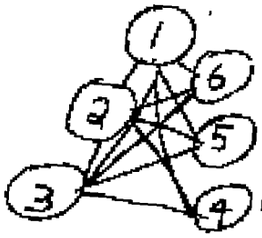
number 1 shakes 4 people
number 2 shakes 3 People
number 3 shakes 2 People
Number 4 shakes 1 People
and number 5 shakes 0 People



A 4+3+2+1=10

J At first I made a chart using five bubbles numbering each. I did this to find the answer number one would shake 4 not shaking his own hand. I then drawed four lines to 4, 3, 2 and 1. Then I did the exact same routine w/ 4, 3 and 2 and 1. I did not shake any ones hand. Then I counted the lines and added them and it equeled 10. That is how many hand shakes in all.

B



1 shook 5 People hands
2 shook 4 People hands
3 shook 3 People hands
4 shook 2 People hands
5 shook 1 People hands
6 shook no hands

5+4+3+2+1= 15

M5 **b** Problem Solving and Reasoning: Implementation. The student makes the basic choices involved in planning and carrying out a solution; that is, the student:

- makes up and uses a variety of strategies and approaches to solving problems and uses or learns approaches that other people use, as appropriate.

A B H I Both students used diagrams, lists, and equations.

- makes connections among concepts in order to solve problems.

A B C E F H I Both students made connections between simple number concepts and patterns to solve the problem.

- solves problems in ways that make sense and explains why these ways make sense, e.g., defends the reasoning, explains the solution.

A B C E F H I J

C I noticed a pattern. Since you don't have 6 shake any hands, you write 5, then add 4 then 3 then 2 then 1, and then it equals 15. That pattern helps me to realize not to use a chart on 7 because I can use my way like I did on the pattern. If there where 7 people I would just have to make a math sentence like this:

6+5+4+3+2+1=21

D A way you can always figure it out without a chart is by using a math sentence. Starting at the highest of your number, or the lowest, going up or down then adding all of them together. Remember not to include the person who can't shake his own hand. For exsample if you had 30 people you might start with 29 because 30 can not shake his own hand. Then you would count down to 1. Then add all the numbers together and that would equal the correct answer.

Arithmetic and Number Concepts
Geometry & Measurement Concepts
M3 Function & Algebra Concepts
M4 Statistics & Probability Concepts
M5 Problem Solving & Reasoning
Mathematical Skills & Tools
M7 Mathematical Communication
Putting Mathematics to Work

How Many Handshakes?

Arithmetic and Number Concepts
Geometry& Measurement Concepts
Function & Algebra Concepts M3
Statistics & Probability Concepts M4
Problem Solving & Reasoning M5
Mathematical Skills & Tools
Mathematical Communication M7
Putting Mathematics to Work

M5 c Problem Solving and Reasoning: Conclusion. The student moves beyond a particular problem by making connections, extensions, and/or generalizations; for example, the student:...

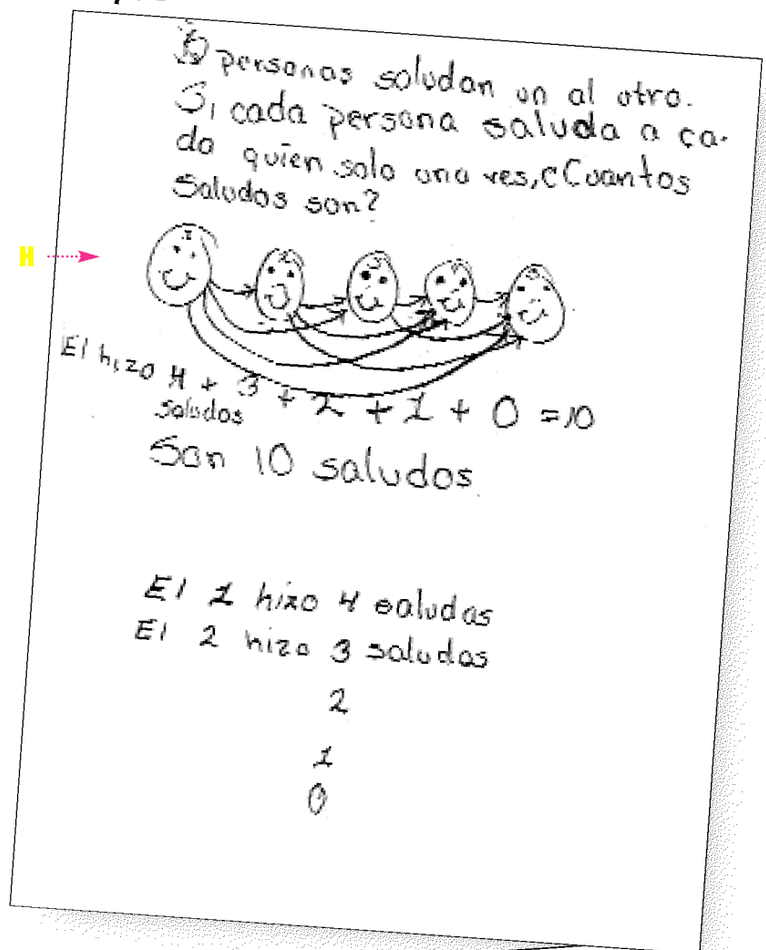
- makes the solution into a general rule that applies to other circumstances.

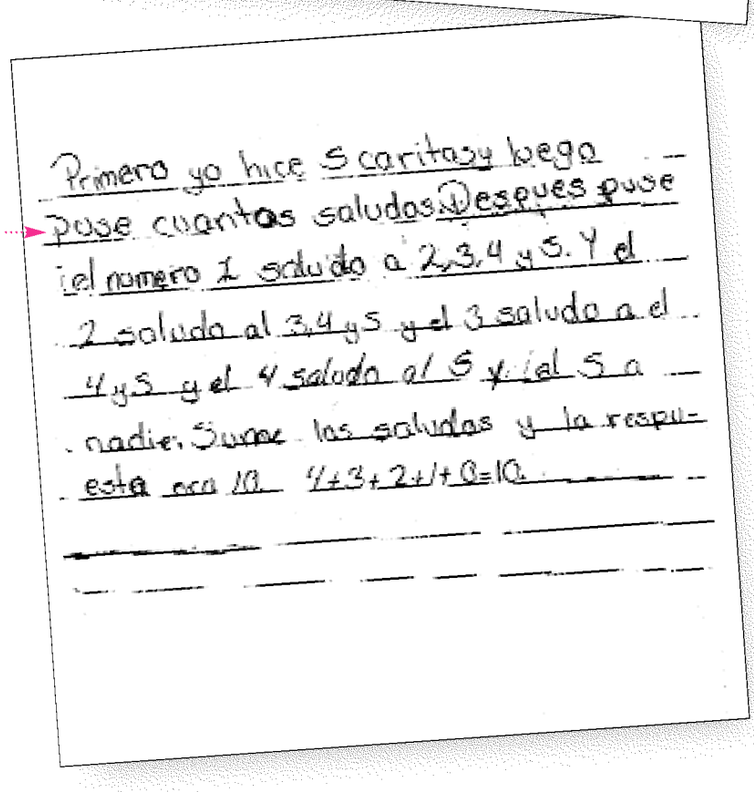
D G Both students went beyond the problem by finding a rule that would work for any number of people.

M7 b Mathematical Communication: The student shows mathematical ideas in a variety of ways, including words, numbers,...pictures,...diagrams....

A B C D E F G H I J

Sample 2





How Many Handshakes?

Sample 2 continued

7 personas saludan a uno al otro solamente una vez. ¿Cuantos saludos son?

I → 6+5+4+3+2+1+0=21

El 1 hizo 6 saludos
El 2 hizo 5 saludos
El 3 hizo 4 saludos
El 4 hizo 3 saludos
El 5 hizo 2 saludos
El 6 hizo 1 saludo
El 7 hizo 0 saludos

F → Primero puse 7 caritas y luego puse cuantos saludos. Despues puse el numero 1 saludo 2,3,4,5,6 y 7 y tambien hise lo mismo con 2,3,4,5,6 y 7. Despues los sume: 6+5+4+3+2+1+0=21

Sample 2 Translation

E → First I made 5 faces and then I put how many handshakes. After that I put that number 1 shakes the hand of 2,3,4, and 5. And number 2 shakes 3.4. and 5 and number 3 shakes 4 and 5 and number 4 shakes 5 and number 5 shakes nobody's hand. I added the handshakes and the answer is 10.
4+3+2+1+0 =10

F → First I put 7 faces and then I put how many handshakes. I put that number 1 shakes the hand of 2,3,4,5,6 and 7 and also I did the same with 2,3,4,5,6 and 7. After that I added:
6+5+4+3+2+1+0 = 21

G → It is easy. I only have to start with the number that is one less and add the rest. For example: If there are 39 I have to start with 38.

G → Es fácil. Sólo tengo que empeza con el número que es uno menos y sumar los demas Por ejemplo:
si son 39 tengo que empecar con el 38.

M7 ‹ Mathematical Communication: The student explains solutions to problems clearly and logically, and supports solutions with evidence, in both written and oral work.

A B C D E F G H I J

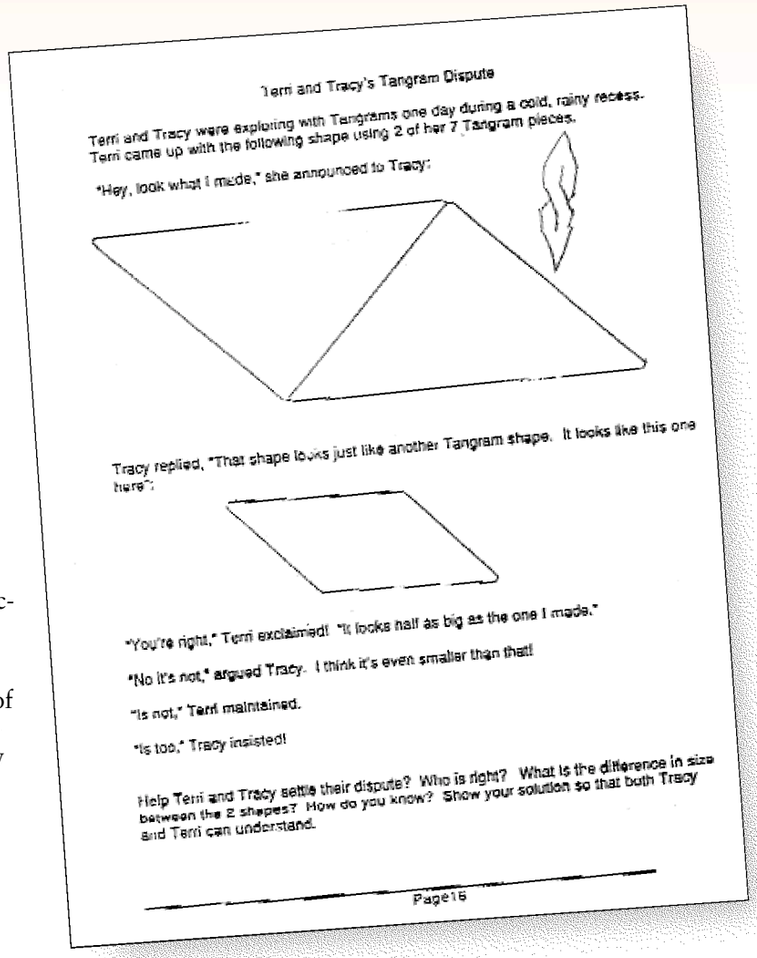
Work Sample & Commentary: *Tangram Dispute*

Arithmetic and Number Concepts M1
Geometry & Measurement Concepts M2
Function & Algebra Concepts
Statistics & Probability Concepts
Problem Solving & Reasoning M5
Mathematical Skills & Tools M6
Mathematical Communication M7
Putting Mathematics to Work

The quotations from the Mathematics performance descriptions in this commentary are excerpted. The complete performance descriptions are shown on pages 60-65.

The task

The teacher gave students the prompt (the diagrams were the size of tangram pieces) illustrated here.



Terri and Tracy's Tangram Dispute

Terri and Tracy were exploring with Tangrams one day during a cold, rainy recess. Terri came up with the following shape using 2 of her 7 Tangram pieces.

"Hey, look what I made," she announced to Tracy:

Tracy replied, "That shape looks just like another Tangram shape. It looks like this one here":

"You're right," Terri exclaimed! "It looks half as big as the one I made."

"No it's not," argued Tracy. I think it's even smaller than that!

"Is not," Terri maintained.

"Is too," Tracy insisted!

Help Terri and Tracy settle their dispute? Who is right? What is the difference in size between the 2 shapes? How do you know? Show your solution so that both Tracy and Terri can understand.

Page 16

Circumstances of performance

This sample of student work was produced under the following conditions:

alone	in a group
in class	as homework
with teacher feedback	with peer feedback
timed	opportunity for revision

Students had previously worked with tangrams. They had begun to study area relationships and fractions.

This student was preparing a New Standards Elementary Mathematics Portfolio and was aware of the requirements for the portfolio. In providing evidence of problem solving work, students using New Standards mathematics portfolios must show that they can go beyond the problem and make connections, extensions or generalizations (M5 c).

This work sample illustrates a standard-setting performance for the following parts of the standards:

- **M1 d Arithmetic and Number Concepts: Describe and compare quantities by using simple fractions.**
- **M2 e Geometry and Measurement Concepts: Solve problems by showing relationships between and among figures.**
- **M5 a Problem Solving and Reasoning: Formulation.**
- **M5 b Problem Solving and Reasoning: Implementation.**
- **M5 c Problem Solving and Reasoning: Conclusion.**
- **M6 e Mathematical Skills and Tools: Refer to geometric shapes and terms correctly.**
- **M7 b Mathematical Communication: Show mathematical ideas in a variety of ways.**
- **M7 c Mathematical Communication: Explain solutions to problems clearly and logically.**

What the work shows

M1 d Arithmetic and Number Concepts: The student describes and compares quantities by using concrete and real world models of simple fractions; that is:

- **finds simple parts of wholes.**

A The student explained and showed in a model that there can be four equal (parallelogram) parts within the whole (parallelogram).

- **uses drawings, diagrams, or models to show what the numerator and denominator mean, including when adding like fractions, e.g., ⅛ + ⅝, or when showing that ¾ is more than ⅜.**

B C The student used two models to show that one small parallelogram is equivalent to one out of four equal parts of the whole parallelogram, i.e., that "Tracy's shape was ¼ not a ½ of Terri's shape."

Tangram Dispute

M2 e Geometry and Measurement Concepts: The student solves problems by showing relationships between and among figures, e.g., using congruence and similarity….

D The student explained and modeled that "…one big shape can equal (i.e., be congruent to) four little of the same (i.e., a similar) shape."

M5 a Problem Solving and Reasoning: Formulation. Given the basic statement of a problem situation, the student:

- **makes the important decisions about the approach, materials, and strategies to use, i.e., does not merely fill in a given chart, use a pre-specified manipulative, or go through a predetermined set of steps.**

A B C D Given a problem situation with many possible approaches, the student decided what materials to use (pattern blocks and tiles) and what approach to follow.

- **uses previously learned strategies, skills, knowledge, and concepts to make decisions.**

A The student used knowledge about the relationship between simple parts and wholes to decide who was correct in the dispute.

- **uses strategies, such as using manipulatives or drawing sketches, to model problems.**

B C D

M5 b Problem Solving and Reasoning: Implementation. The student makes the basic choices involved in planning and carrying out a solution; that is, the student:

- **makes up and uses a variety of strategies and approaches to solving problems and uses or learns approaches that other people use, as appropriate.**

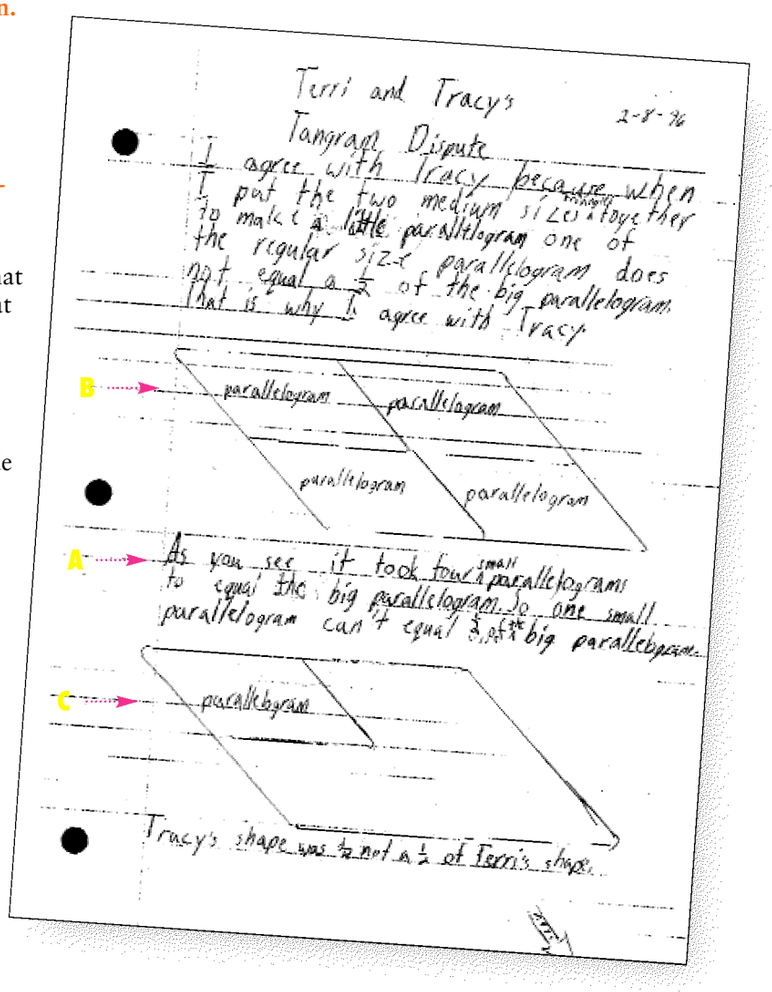
B C The student used fractional and geometric strategies to solve the problem.

- **makes connections among concepts in order to solve problems.**

A B C D The student made connections between number and geometry concepts to solve the problem.

- **solves problems in ways that make sense and explains why these ways make sense, e.g., defends the reasoning, explains the solution.**

A B C



Tangram Dispute

Arithmetic and Number Concepts M1
Geometry& Measurement Concepts M2
Function & Algebra Concepts
Statistics & Probability Concepts
Problem Solving & Reasoning M5
Mathematical Skills & Tools M6
Mathematical Communication M7
Putting Mathematics to Work

M5 c Problem Solving and Reasoning: Conclusion. The student moves beyond a particular problem by making connections, extensions, and/or generalizations; for example, the student:…

- **explains how the problem is similar to other problems he or she has solved.…**

D The student went beyond the problem by illustrating that the same approach and concepts can be used in a different situation.

M6 e Mathematical Skills and Tools: The student refers to geometric shapes and terms correctly with concrete objects or drawings, including triangle,…parallelogram.…

B The student referred correctly to the parallelogram, which is not mentioned by name in the task.

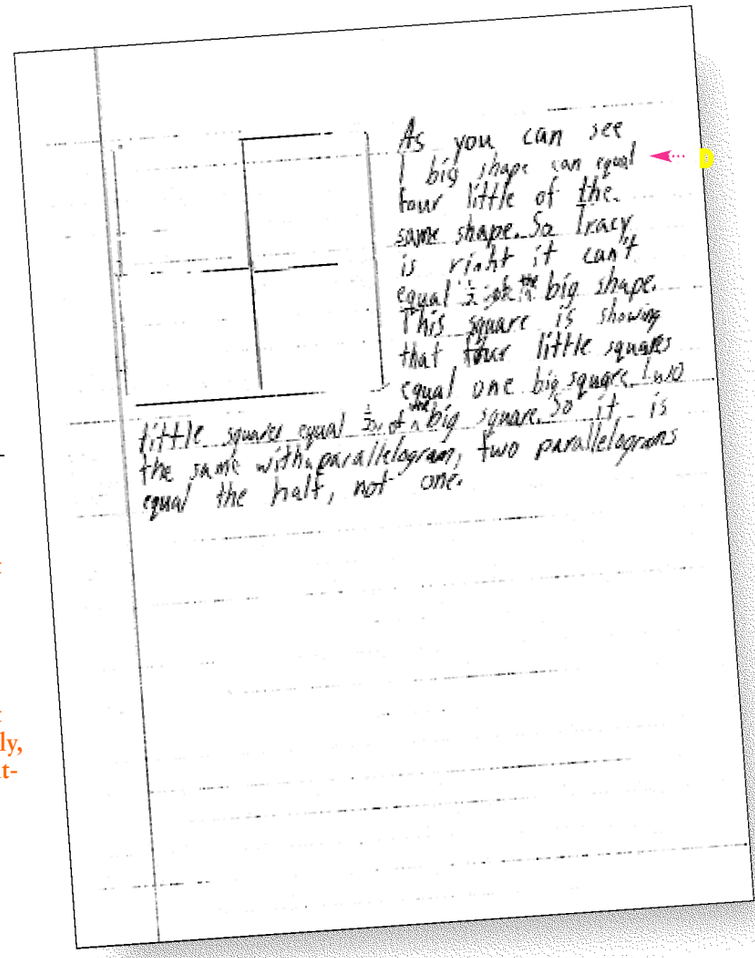
M7 b Mathematical Communication: The student shows mathematical ideas in a variety of ways, including words, numbers,…pictures.…

A B C D

M7 c Mathematical Communication: The student explains solutions to problems clearly and logically, and supports solutions with evidence, in both written and oral work.

A The solution is explained clearly.

B C The conclusion is supported by diagrammatic evidence.



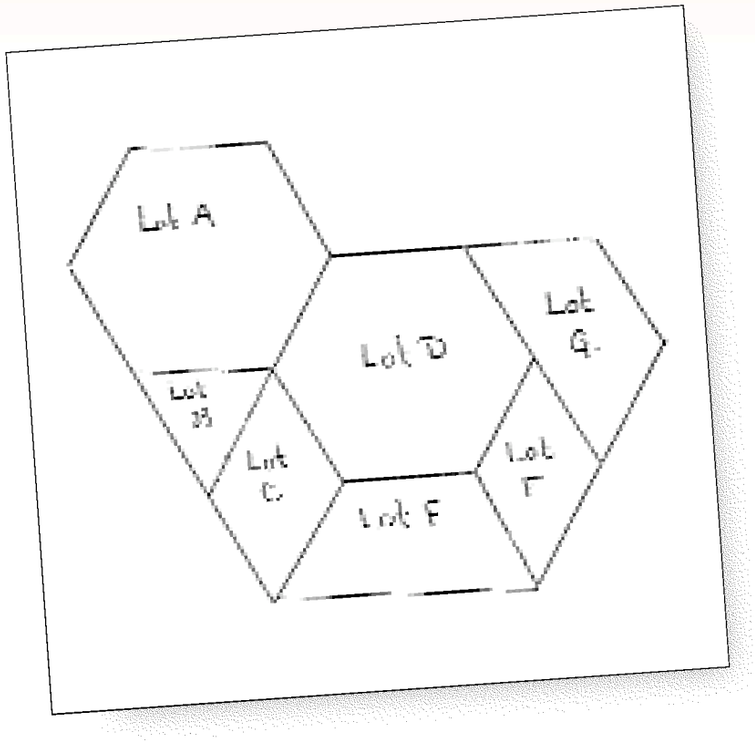
Work Sample & Commentary: *Feverish Freddy*

The task

The teacher gave students the following prompt:

Freddy, a very precise real estate appraiser, was sent to appraise some lots on a local property. Freddy appraised lot A for $88,000, but then came down with the flu and was unable to determine the fair market value of the other lots. Help Freddy determine the values so he does not lose his job!

Using the value Freddy determined for lot A, figure the fair value of each of the other lots in relation to lot A. Write a report to Freddy informing him of your mathematical determinations. Show Freddy how you reached your conclusion, so he will have faith in your determination. Represent your determination mathematically, and use appropriate math language and representations to support your conclusion. Share with Freddy any mathematically relevant observations you make, as well as any recommendations.



In the actual task, this diagram matched the size of pattern blocks.

This work sample illustrates a standard-setting performance for the following parts of the standards:

- M1 a Arithmetic and Number Concepts: Add, subtract, multiply, and divide whole numbers.
- M1 d Arithmetic and Number Concepts: Describe and compare quantities by using simple fractions.
- M2 e Geometry and Measurement Concepts: Solve problems by showing relationships between and among figures.
- M3 d Function and Algebra Concepts: Use letters, boxes, or other symbols to stand for any number, measured quantity, or object in simple situations.
- M5 a Problem Solving and Reasoning: Formulation.
- M5 b Problem Solving and Reasoning: Implementation.
- M5 c Problem Solving and Reasoning: Conclusion.
- M6 e Mathematical Skills and Tools: Refer to geometric shapes and terms correctly.
- M6 g Mathematical Skills and Tools: Read, create, and represent data.
- M6 h Mathematical Skills and Tools: Use manipulatives and calculators to achieve solutions.
- M7 b Mathematical Communication: Show mathematical ideas in a variety of ways.

Circumstances of performance

This sample of student work was produced under the following conditions:

alone	in a group
in class	as homework
with teacher feedback	with peer feedback
timed	opportunity for revision

Students had prior experience working with pattern blocks and were familiar with their fractional equivalents (with the hexagon equal to one whole).

What the work shows

M1 a Arithmetic and Number Concepts: The student adds, subtracts, multiplies, and divides whole numbers, with and without calculators; that is:...

- analyzes problem situations and contexts in order to figure out when to add, subtract, multiply, or divide.

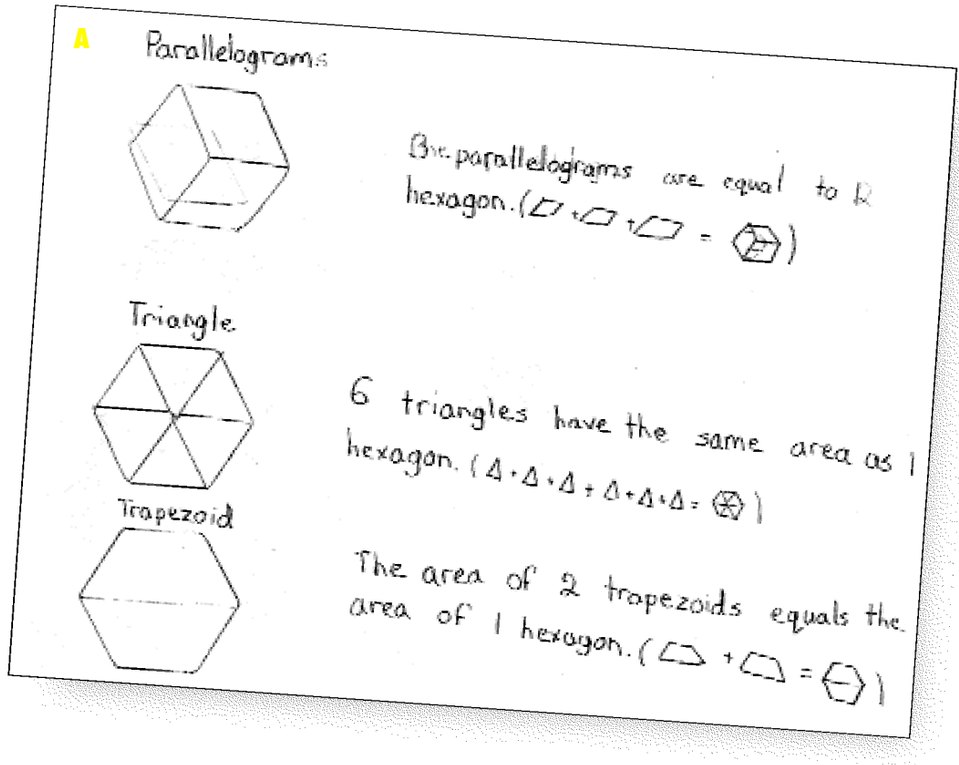
A The student, given a problem situation with many possible approaches, figured out which arithmetic operations to use in order to come to a correct solution (evident throughout the work, without citing a specific operation).

- solves arithmetic problems by relating addition, subtraction, multiplication, and division to one another.

B

The quotations from the Mathematics performance descriptions in this commentary are excerpted. The complete performance descriptions are shown on pages 60-65.

Feverish Freddy



Arithmetic and Number Concepts M1
Geometry & Measurement Concepts M2
Function & Algebra Concepts M3
Statistics & Probability Concepts
Problem Solving & Reasoning M5
Mathematical Skills & Tools M6
Mathematical Communication M7
Putting Mathematics to Work

M1 d Arithmetic and Number Concepts: The student describes and compares quantities by using concrete and real world models of simple fractions; that is:

- **finds simple parts of wholes….**

A

M2 e Geometry and Measurement Concepts: The student solves problems by showing relationships between and among figures, e.g., using congruence….

C The student compared congruent fractional parts of hexagons to determine the relative value of each part.

M3 d Function and Algebra Concepts: The student uses letters, boxes, or other symbols to stand for any number, measured quantity, or object in simple situations with concrete materials, i.e., demonstrates understanding and use of a beginning concept of a variable.

B

M5 a Problem Solving and Reasoning: Formulation. Given the basic statement of a problem situation, the student:

- **makes the important decisions about the approach, materials, and strategies to use, i.e., does not merely fill in a given chart, use a pre-specified manipulative, or go through a predetermined set of steps.**

A The student decided which approach (finding simple parts of hexagons and determining their relative value) and material (pattern blocks) to use.

- **uses previously learned strategies, skills, knowledge, and concepts to make decisions.**

A C The student used knowledge of the fractional equivalents among pattern block pieces to make decisions about the value of each piece (i.e., each lot).

- **uses strategies, such as using manipulatives or drawing sketches, to model problems.**

A

M5 b Problem Solving and Reasoning: Implementation. The student makes the basic choices involved in planning and carrying out a solution; that is, the student:

- **makes up and uses a variety of strategies and approaches to solving problems….**

A B The student used pictorial, geometric, narrative, and arithmetic approaches in the solution.

- **makes connections among concepts in order to solve problems.**

A C D The student connected the fractional concepts with geometric concepts of congruence and whole number concepts of value to solve the problem.

- **solves problems in ways that make sense and explains why these ways make sense, e.g., defends the reasoning, explains the solution.**

C The student explained the process used to reach the solution, without stating explicitly how the exact dollar value of each lot was derived.

E The work provides evidence of making sense of what to do with the cents that were left over.

Feverish Freddy

E On lot B,C and F there was a certain amount of cents. I didn't add them to the main number because I didn't think it was important.

C On the lots 3 lot Fs or Gs would equal 1 lot A or D. 1 lot G and 1 lot E is equal to 1 lot A or D. Lot B is equal to $\frac{1}{2}$ of lot F or C. 3 lot Bs have the same area as 1 lot F or G.

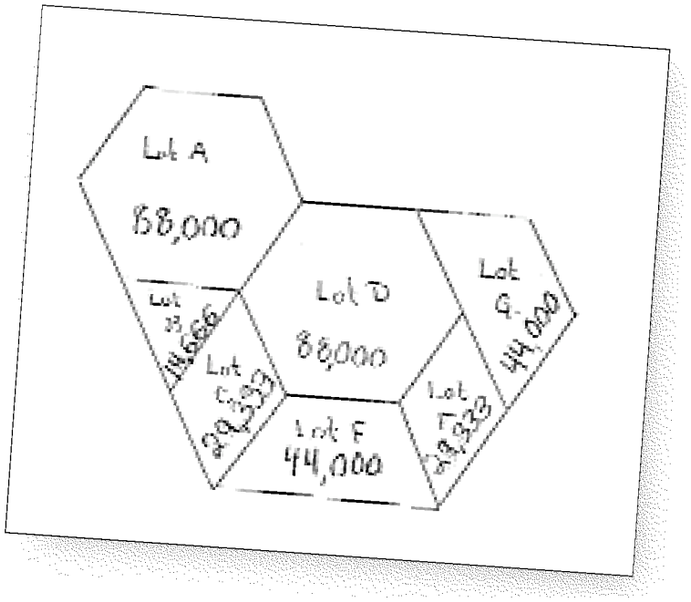
$3F = A$
$E + G = A$
$B = \frac{F}{2}$

Each of the letters equal 1 lot put into a number sentence

B
$3B = G$
$2F + 2B = A$
$F + B = E$
$A - 2F - B = B$
$D - 3B = G$
$G + C + B = D$

D

Shape of lot	Cost	Lot Letter(s)	total cost
hexagon	88,000	A and D	176,000
parallelogram	29,333	C and F	58,666
triangle	14,666	B	14,666
trapezoid	44,000	E and G	88,000



M5 c Problem Solving and Reasoning: Conclusion. The student moves beyond a particular problem by making connections, extensions, and/or generalizations….

B The student extended the problem by listing algebraic equations that show the fractional relationships among the lots. Expressing mathematical relationships algebraically goes beyond what most elementary students know and can be expected to do.

M6 e Mathematical Skills and Tools: The student refers to geometric shapes and terms correctly with concrete objects or drawings, including triangle,…parallelogram….

A D

M6 g Mathematical Skills and Tools: The student reads, creates, and represents data on…charts, tables, diagrams….

A D The student created a chart and diagram to organize and represent information.

M6 h Mathematical Skills and Tools: The student uses…manipulatives, calculators,…as appropriate, to achieve solutions.

A D The student used pattern blocks and a calculator (in "Cost" and "Total cost") to achieve a solution. The teacher verified that the student used a calculator.

M7 b Mathematical Communication: The student shows mathematical ideas in a variety of ways, including words, numbers, symbols, pictures, charts, graphs, tables, diagrams, and models.

A B C D

Work Sample & Commentary: Counting on Frank

Arithmetic and Number Concepts M1
Geometry & Measurement Concepts M2
Function & Algebra Concepts
Statistics & Probability Concepts
Problem Solving & Reasoning M5
Mathematical Skills & Tools M6
Mathematical Communication M7
Putting Mathematics to Work

The quotations from the Mathematics performance descriptions in this commentary are excerpted. The complete performance descriptions are shown on pages 60-65.

This work sample is also included among the work samples that illustrate the English Language Arts performance standards. See page 46.

The task

Students were asked to read *Counting on Frank* by Rod Clement and to write a letter to the author commenting on at least one example of the mathematical claims made.

Circumstances of performance

This sample of student work was produced under the following conditions:

alone	in a group
in class	as homework
with teacher feedback	with peer feedback
timed	opportunity for revision

Four students discussed the mathematics in *Counting on Frank* in depth before they wrote their analyses. The students spent class time as well as many recesses figuring out how to test the claims in the book. For example, the students discussed how to use the classroom sink to test the claim in the book about the bathroom filling up with water. The students worked together to develop strategies for testing the claims about the peas and the ball-point pen. The teacher encouraged the students' discussions, and provided time and materials in school for them to work out their reasoning. The students met outside of class to work together on the writing. Students completed the writing individually, at home.

The students completed this activity near the end of the school year. Earlier in the year, the students had studied area and perimeter concepts in depth, as well as applications of multiplication and division in problem solving activities.

This work sample represents a standard-setting performance for the following parts of the standards:

- **M1 a** Arithmetic and Number Concepts: Add, subtract, multiply, and divide whole numbers.
- **M1 b** Arithmetic and Number Concepts: Demonstrate understanding of the base ten place value system and use this knowledge to solve arithmetic tasks.
- **M1 c** Arithmetic and Number Concepts: Estimate, approximate, round off, use landmark numbers, or use exact numbers, as appropriate, in calculations.
- **M1 d** Arithmetic and Number Concepts: Describe and compare quantities by using concrete and real world models of simple fractions.
- **M1 e** Arithmetic and Number Concepts: Describe and compare quantities by using simple decimals.
- **M1 f** Arithmetic and Number Concepts: Describe and compare quantities by using whole numbers up to 10,000.
- **M2 g** Geometry and Measurement Concepts: Use basic ways of estimating and measuring the size of figures and objects in the real world.
- **M2 i** Geometry and Measurement Concepts: Select and use units for estimating and measuring quantities.
- **M2 j** Geometry and Measurement Concepts: Carry out simple unit conversions.
- **M5 a** Problem Solving and Reasoning: Formulation.
- **M5 b** Problem Solving and Reasoning: Implementation.
- **M5 c** Problem Solving and Reasoning: Conclusion.
- **M6 a** Mathematical Skills and Tools: Add, subtract, multiply, and divide whole numbers correctly.
- **M6 b** Mathematical Skills and Tools: Estimate numerically and spatially.
- **M6 c** Mathematical Skills and Tools: Measure accurately.
- **M6 d** Mathematical Skills and Tools: Compute time.
- **M6 f** Mathematical Skills and Tools: Use +, -, x, ÷, /,...and . (decimal point) correctly in number sentences and expressions.
- **M6 h** Mathematical Skills and Tools: Use recall, mental computations, pencil and paper, measuring devices, manipulatives, calculators and advice from peers, as appropriate, to achieve solutions.
- **M7 a** Mathematical Communication: Use appropriate mathematical terms, vocabulary, and language.
- **M7 b** Mathematical Communication: Show mathematical ideas in a variety of ways.
- **M7 c** Mathematical Communication: Explain solutions to problems clearly and logically.
- **M7 d** Mathematical Communication: Consider purpose and audience when communicating about mathematics.
- **M7 e** Mathematical Communication: Comprehend mathematics from reading assignments and from other sources.

Counting on Frank

What the work shows

M1 a Arithmetic and Number Concepts: The student adds, subtracts, multiplies, and divides whole numbers, with and without calculators; that is:…

- **multiplies, i.e.,…uses simple rates.**

A B C D E F The consistent and effective use of multiplication throughout the student work shows a deep understanding at the elementary level.

- **divides, i.e., puts things into groups, shares equally; calculates simple rates.**

G H I J

- **analyzes problem situations and contexts in order to figure out when to add, subtract, multiply, or divide….**

K L M The student successfully figured out how and when to use arithmetic in these multi-step problems. The analysis is particularly strong because the situations were general claims made by a character in a book, i.e., there was no hint about how to proceed to test the claims.

- **solves arithmetic problems by relating addition, subtraction, multiplication, and division to one another.**

E The student added to complete a multiplication problem. The teacher also verified that the student divided 700 by 4 and multiplied the answer by 3 with a calculator to figure out that "three quarters of 700 equals 525."

- **computes answers mentally….**

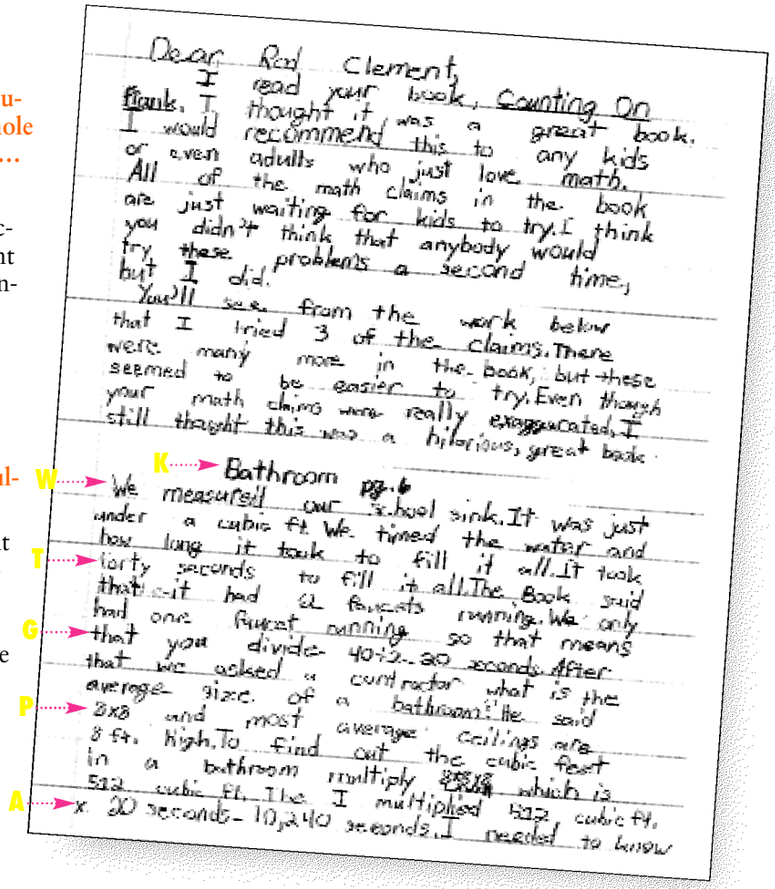
D G N O The student, as verified by the teacher, computed some of the arithmetic mentally, here and in other places in the work.

M1 b Arithmetic and Number Concepts: The student demonstrates understanding of the base ten place value system and uses this knowledge to solve arithmetic tasks; that is:…uses knowledge about ones, tens, hundreds, and thousands to figure out answers to multiplication and division tasks, e.g., 36 x 10, 18 x 100, 7 x 1,000, 4,000 ÷ 4.

D E N

M1 c Arithmetic and Number Concepts: The student estimates, approximates, rounds off, uses landmark numbers, or uses exact numbers, as appropriate, in calculations.

J N O P Q R The student used a well-developed sense of when it is appropriate to round off or estimate in these situations. Elsewhere, the student successfully used exact numbers in calculations.



Dear Red Clement,
I read your book, Counting On Frank. I thought it was a great book. I would recommend this to any kids or even adults who just love math. All of the math claims in the book are just waiting for kids to try. I think you didn't think that anybody would try these problems a second time, but I did. You'll see from the work below that I tried 3 of the claims. There were many more in the book, but these seemed to be easier to try. Even though your math claims were really exaggerated, I still thought this was a hilarious, great book.

K → Bathroom pg. 6

W → We measured our school sink. It was just under a cubic ft. We timed the water and how long it took to fill it all. It took

T → forty seconds to fill it all. The Book said that it had 2 faucets running. We only had one faucet running so that means

G → that you divide 40÷2. 20 seconds. After that we asked a contractor what is the average size of a bathroom. He said

P → 8x8 and most average ceilings are 8 ft. high. To find out the cubic feet in a bathroom multiply which is 512 cubic ft. The I multiplied 512 cubic ft.

A → x 20 seconds = 10,240 seconds. I needed to know

M1 d Arithmetic and Number Concepts: The student describes and compares quantities by using concrete and real world models of simple fractions; that is, finds simple parts of wholes….

E The student used coffee cups and measured out 4¾ cups. The computation ("three quarters of 700 is 525") is advanced for this level.

M1 e Arithmetic and Number Concepts: The student describes and compares quantities by using simple decimals….

J The student took part of the decimal answer to a division problem (13.13) and used the nearest whole number (13) to make sense of the given situation, i.e., bags of peas.

Counting on Frank

Arithmetic and Number Concepts M1
Geometry & Measurement Concepts M2
Function & Algebra Concepts
Statistics & Probability Concepts
Problem Solving & Reasoning M5
Mathematical Skills & Tools M6
Mathematical Communication M7
Putting Mathematics to Work

M1 f Arithmetic and Number Concepts: The student describes and compares quantities by using whole numbers up to 10,000; that is, connects ideas of quantities to the real world….

K L M The student called the author's bluff by actually connecting the ideas in the book to the real world.

A C J In each problem, the student dealt successfully with quantities larger than 10,000 (10,240 seconds; 43,680 peas; 13,500 feet). The clarity and thoroughness of communication provides evidence of understanding of these quantities and represents a strong performance for this level.

M2 g Geometry and Measurement Concepts: The student uses basic ways of estimating and measuring the size of figures and objects in the real world, including length, width, perimeter, and area.

P The student went beyond the standards by measuring the volume of the bathroom in cubic feet. The teacher verified that the students had constructed the formula for area themselves earlier in the year through conceptual activities. The students' motivation to test the claim about the bathroom led them to a discussion of how to measure the size of the sink and the bathroom, and they constructed the idea of "layers" of length times width. The teacher then supplied them with the formula L x W x H.

Q The student demonstrated understanding of area by listing some of the "millions of possibilities…that are close" for 264 square feet. The student also demonstrated understanding by applying the concept of area to the size of a bathroom floor, and noticing that an area of 264 square feet would be "huge."

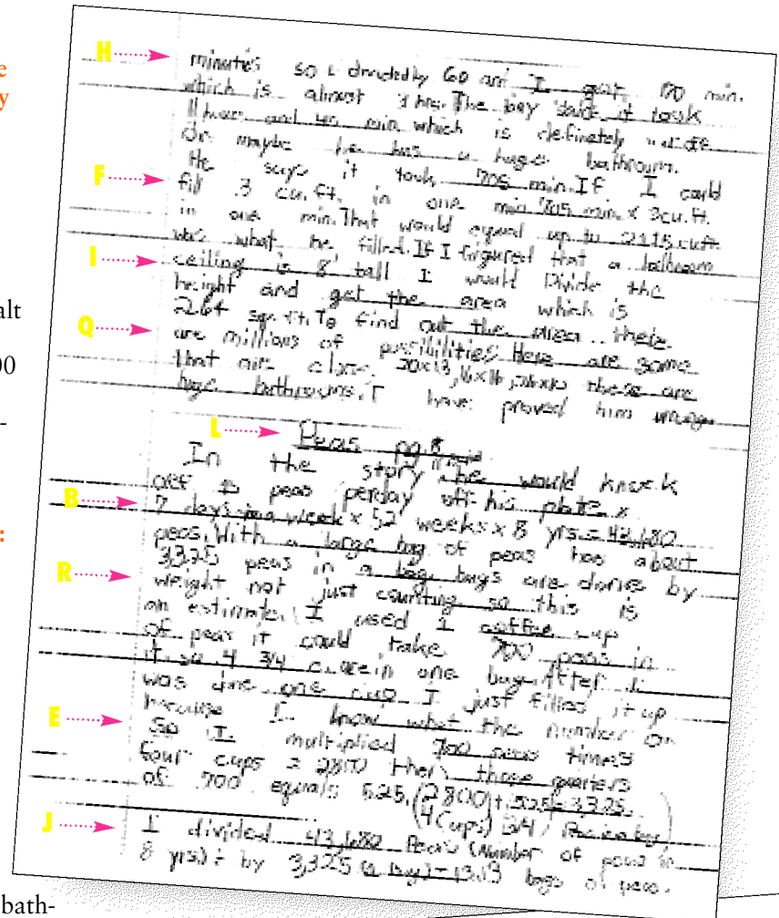
D S The student measured the length of the pen and the length of the line and used these measurements to solve the problem.

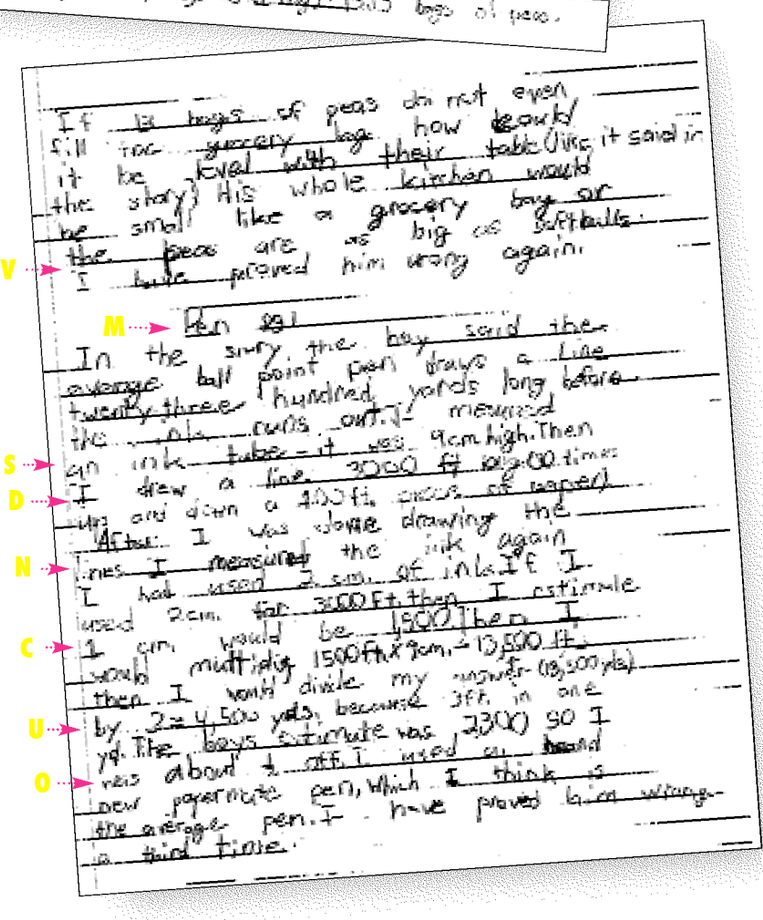
M2 i Geometry and Measurement Concepts: The student selects and uses units, both formal and informal as appropriate, for estimating and measuring quantities such as…length, area, volume, and time.

D S The student used centimeters to measure the pen, and feet to measure the line.

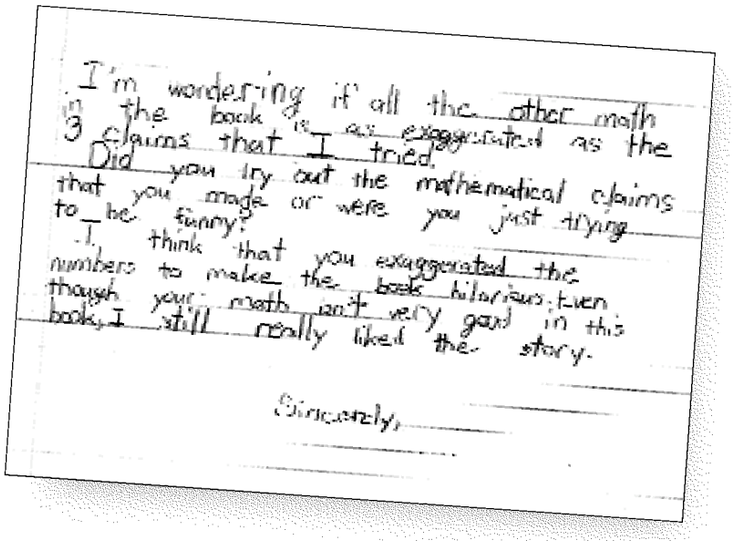
R The student used a coffee cup to measure volume.

T The student used seconds to measure time.





Counting on Frank


I'm wondering if all the other math in the book is as exaggerated as the 3 claims that I tried.
Did you try out the mathematical claims that you made or were you just trying to be funny?
I think that you exaggerated the numbers to make the book hilarious. Even though your math isn't very good in this book, I still really liked the story.
Sincerely,

M2 j Geometry and Measurement Concepts: The student carries out simple unit conversions, such as between cm and m, and between hours and minutes.

B H U The student converted peas per day to peas per year, seconds to minutes, minutes to hours, and feet to yards.

M5 a Problem Solving and Reasoning: Formulation. Given the basic statement of a problem situation, the student:

- **makes the important decisions about the approach, materials, and strategies to use, i.e., does not merely fill in a given chart, use a pre-specified manipulative, or go through a predetermined set of steps.**

K L M The student made the decisions to use the sink, the peas, the coffee cup, the ballpoint pen, and the 100 feet long piece of paper.

- **uses previously learned strategies, skills, knowledge, and concepts to make decisions.**

K L M Throughout the three problems, the student drew upon knowledge and skills related to multiplication, division, and measurement (i.e., length, area, and volume).

- **uses strategies, such as using manipulatives or drawing sketches, to model problems.**

K L M The student devised effective strategies, such as timing how long the water took to fill the sink, using the coffee cup to measure the peas, and measuring the decrease in length of the ink in the pen.

M5 b Problem Solving and Reasoning: Implementation. The student makes the basic choices involved in planning and carrying out a solution; that is, the student:

- **makes up and uses a variety of strategies and approaches to solving problems….**

K L M

- **makes connections among concepts in order to solve problems.**

K L M There are many connections between arithmetic and measurement throughout the work.

- **solves problems in ways that make sense and explains why these ways make sense, e.g., defends the reasoning, explains the solution.**

O Q V Statements like, "His whole kitchen would be small like a grocery bag or the peas are as big as softballs," show that the student made sense of the problem. Throughout the work, the student impressively, consistently, and accurately explicates the reasoning used.

M5 c Problem Solving and Reasoning: Conclusion. The student moves beyond a particular problem by making connections, extensions, and/or generalizations….

L M The assignment asked the student to focus on one claim made in the book. The student chose to extend the assignment by analyzing and writing about more than one claim. This is a reasonable extension for elementary school level.

M6 a Mathematical Skills and Tools: The student adds, subtracts, multiplies, and divides whole numbers correctly; that is:

- **knows single digit addition, subtraction, multiplication, and division facts.**

D E G N The student showed a command of basic facts, which was needed to carry out these and other calculations.

- **adds…numbers with several digits.**

E

- **multiplies and divides numbers with one or two digits.**

G H

- **multiplies and divides three digit numbers by one digit numbers.**

E

M6 b Mathematical Skills and Tools: The student estimates numerically and spatially.

N O Q R

M6 c Mathematical Skills and Tools: The student measures length, area,…height,…and volume accurately in both the customary and metric systems.

D N S W The teacher verified the accuracy of the measurements.

Counting on Frank

M6 d Mathematical Skills and Tools: The student computes time (in hours and minutes)....

H

M6 f Mathematical Skills and Tools: The student uses +, -, x, ÷, /,...and . (decimal point) correctly in number sentences and expressions.

A B C E F G J P Q

M6 h Mathematical Skills and Tools: The student uses recall, mental computations, pencil and paper, measuring devices,...manipulatives, calculators...and advice from peers, as appropriate, to achieve solutions....

K L M The student drew upon many resources to complete the assignment.

M7 a Mathematical Communication: The student uses appropriate mathematical terms, vocabulary, and language, based on prior conceptual work.

K L M The student used many mathematical terms appropriately, including "estimate," "measure," "average," "multiply," "divide," "seconds," "minutes," "area," "long," "high," "equal," "cm," "ft."

P The use of the term "cubic feet" and the formula for volume go beyond what is required at this level.

U The student wrote "yds" after 13,500, but this mislabeling did not interfere with the correct mathematics in the work.

M7 b Mathematical Communication: The student shows mathematical ideas in a variety of ways, including words, numbers, and symbols....

K L M

M7 c Mathematical Communication: The student explains solutions to problems clearly and logically, and supports solutions with evidence, in both oral and written work.

K L M

M7 d Mathematical Communication: The student considers purpose and audience when communicating about mathematics.

K L M The student focused consistently on the purpose of disputing the claims in the book and wrote directly to the author.

M7 e Mathematical Communication: The student comprehends mathematics from reading assignments and from other sources.

K L M The student even comprehended mathematical information from a contractor about the average size of a bathroom.

The student misspelled a few words (e.g., "definetely") and made some grammatical mistakes (e.g., capitalizing "book" in the middle of the sentence). The teacher did not require the students to fix the errors.

Work Sample & Commentary: *School Uniforms Project*

The task

The teacher asked students to come up with a question for a data study that they would then carry out.

This student chose to survey the children of her school about their opinions on the following questions:

Would you like to have uniforms at this school?

What colors would you like the uniforms to be:

- white and red?
- white and magenta?
- white and blue?

This work sample illustrates a standard-setting performance for the following parts of the standards:

- **M1 a Arithmetic and Number Concepts: Add, subtract, multiply, and divide whole numbers.**
- **M1 c Arithmetic and Number Concepts: Estimate, approximate, round off, use landmark numbers, or use exact numbers in calculations.**
- **M1 d Arithmetic and Number Concepts: Describe and compare quantities by using concrete and real world models of simple fractions.**
- **M1 e Arithmetic and Number Concepts: Describe and compare quantities by using simple decimals.**
- **M4 a Statistics and Probability Concepts: Collect and organize data to answer a question.**
- **M4 b Statistics and Probability Concepts: Display data.**
- **M4 c Statistics and Probability Concepts: Make statements and draw simple conclusions based on data.**
- **M4 d Statistics and Probability Concepts: Gather data about an entire group or by sampling group members to understand the concept of sample.**
- **M5 a Problem Solving and Reasoning: Formulation.**
- **M6 a Mathematical Skills and Tools: Add, subtract, multiply, and divide whole numbers correctly.**
- **M6 g Mathematical Skills and Tools: Read, create, and represent data.**
- **M7 d Mathematical Communication: Consider purpose and audience when communicating about mathematics.**
- **M8 a Putting Mathematics to Work: Data study.**

Circumstances of performance

This sample of student work was produced under the following conditions:

alone	in a group
in class	as homework
with teacher feedback	with peer feedback
timed	opportunity for revision

The student developed and carried out this project with three classmates, but wrote an individual report. The class had not previously worked with collecting survey data.

In this classroom, students had been using New Standards Elementary Mathematics Portfolios. The teacher intended for students to be able to use this data survey to satisfy requirements for the project exhibit of the portfolio. According to the instructions for the project exhibit, students completing a data study should do the following things.

- Think of a situation where data can help you make a decision or recommendation. Describe the situation and the question you will ask.
- Decide on the groups and numbers of people you will survey.
- Predict the results of your study using specific fractions, percents, numbers, etc.
- Collect and organize your data in a chart. You may need to reorganize it.
- Compare your result to your prediction. Write down what was the same or different.
- Organize your data into a chart, graph, or diagram that shows your result.
- Record how your results help you make your recommendation or decision.
- Write a report that includes your decision or recommendation. Include all your work.

This sample was completed in Spanish in a bilingual classroom. The translation was provided by the teacher.

The quotations from the Mathematics performance descriptions in this commentary are excerpted. The complete performance descriptions are shown on pages 60-65.

School Uniforms Project

Arithmetic and Number Concepts M1
Geometry & Measurement Concepts
Function & Algebra Concepts
Statistics & Probability Concepts M4
Problem Solving & Reasoning M5
Mathematical Skills & Tools M6
Mathematical Communication M7
Putting Mathematics to Work M8

What the work shows

M1 a Arithmetic and Number Concepts: The student…divides whole numbers, with and without calculators; that is:

- **divides, i.e., puts things into groups, shares equally; calculates simple rates.**

A

- **analyzes problem situations and contexts in order to figure out when to…divide.**

A The student figured out that it was necessary to divide the number of children in the sample by the number of grades in the school in order to get an equal sampling from each grade.

M1 c Arithmetic and Number Concepts: The student estimates, approximates, rounds off,…as appropriate, in calculations.

B The student made the appropriate choice of rounding off in order to more easily compute 10%.

M1 d Arithmetic and Number Concepts: The student describes and compares quantities by using concrete and real world models of simple fractions; that is:

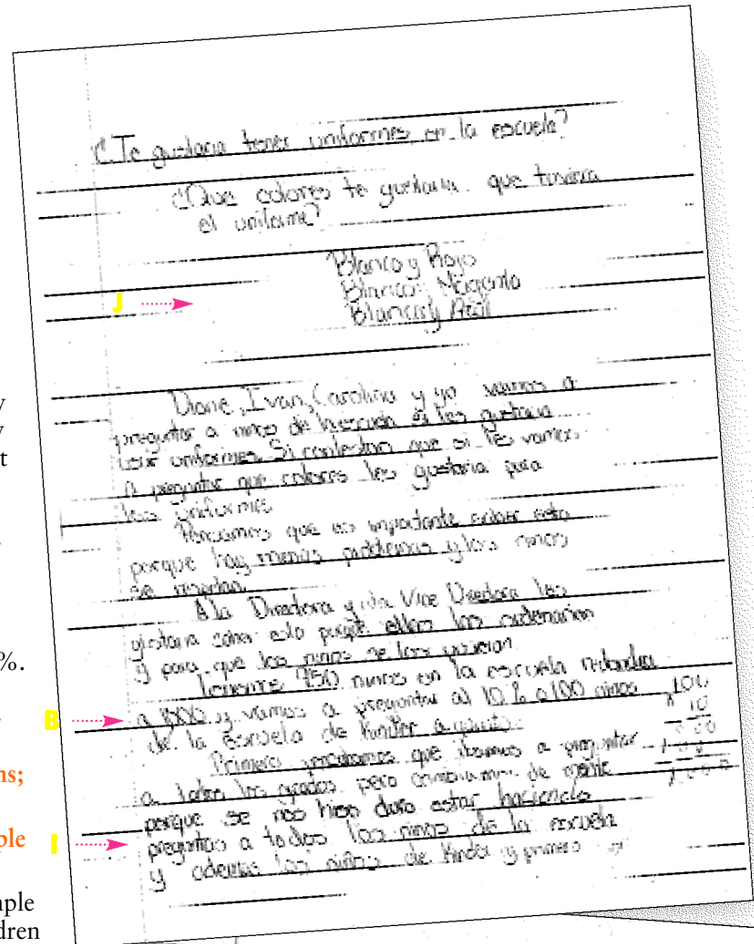
- **uses beginning proportional reasoning and simple ratios….**

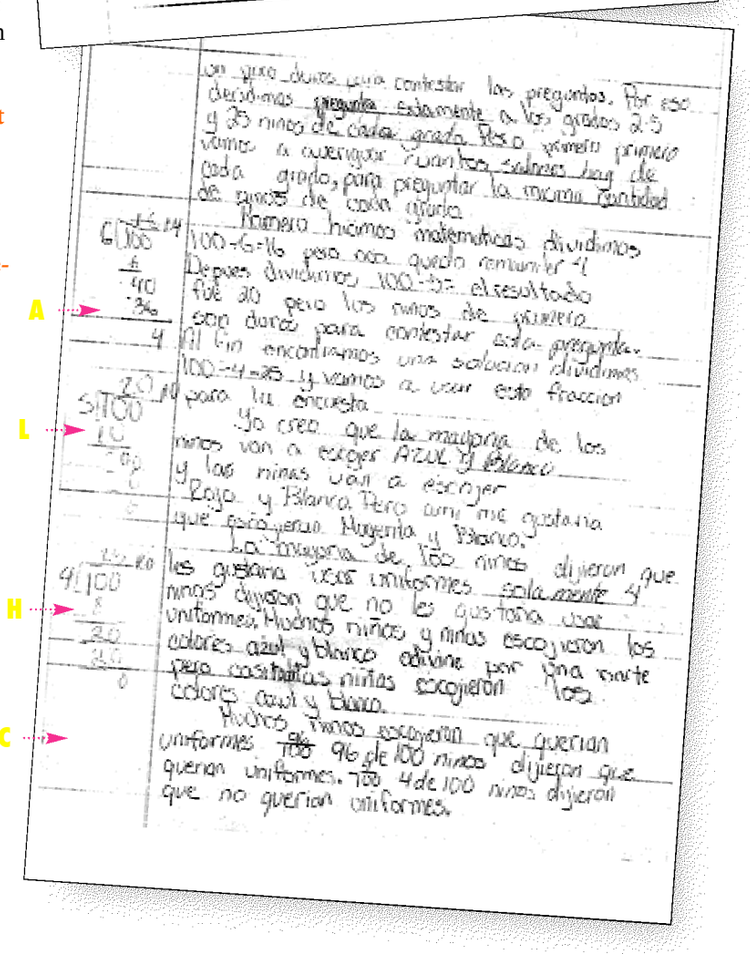
C The student used fractions to represent a simple concept of proportion ("%₀₀…96 out of 100 children said…").

M1 e Arithmetic and Number Concepts: The student describes and compares quantities by using simple decimals; that is:

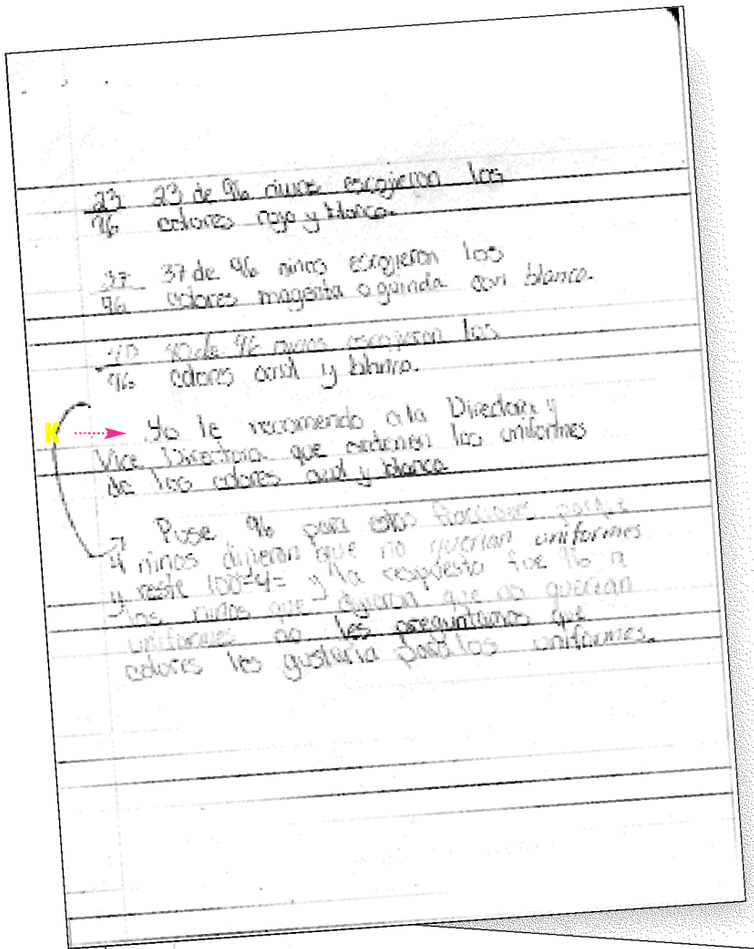
- **recognizes relationships among simple fractions, decimals, and percents, i.e., that ½ is the same as 0.5, and ½ is the same as 50%, with concrete materials, diagrams, and in real world situations….**

B


¿Te gustaría tener uniformes en la escuela?
¿Qué colores te gustaría que tuviera el uniforme?
Blanco y Rojo
Blanco Mugenta
Blanco y Azul
J
Diane, Ivan, Carolina y yo vamos a preguntar a niños de Secundaria si les gustaría usar uniformes. Si contestan que sí les vamos a preguntar que colores les gustaría para los uniformes.
Pensamos que es importante saber esto porque hay muchos padrinos y los niños se mueren.
A la Directora y a la Vice Directora les gustaría saber esto porque ellas son los cabezones y para que los niños no les pasen
Tenemos 750 niños en la escuela redondeando
B 100% y vamos a preguntar al 10% o 100 niños de la Escuela de hombres y mujeres.
Primero pensamos que íbamos a preguntar a todos los grados pero cambiamos de opinión porque se nos hizo duro estar haciendo preguntas a todos los niños de la escuela
I y entonces los niños de kinder y primero


Es que duro para contestar las preguntas. Por eso decidimos preguntar solamente a los grados 2, 5 y 25 niños de cada grado. Pero primero primero vamos a averiguar cuantos salones hay de cada grado, para preguntar la misma cantidad de niños de cada aula.
Primero hicimos matemáticas dividimos 100÷6=16 pero nos quedó remanente 4
Después dividimos 100÷5=20 el resultado fue 20 pero los niños de primero son duros para contestar esta pregunta.
A Al fin encontramos una solución dividimos 100÷4=25 y vamos a usar esta fracción para la encuesta.
Yo creo que la mayoría de los niños van a escoger Azul y Blanco y las niñas van a escoger Rojo y Blanco. Pero a mí me gustaría que escogieran Mugenta y Blanco.
L La mayoría de los niños dijeron que les gustaría usar uniformes solamente 4 niños dijeron que no les gustaría usar uniformes. Hubo niños y niñas escogieron los colores azul y blanco adivine por una parte pero casi todas niñas escogieron los colores azul y blanco.
H
Hubo niños escogieron que querían uniformes 96 de 100 niños dijeron que querían uniformes. 4 de 100 niños dijeron que no querían uniformes.
C

School Uniforms Project



M4 a Statistics and Probability Concepts: The student collects and organizes data to answer a question or test a hypothesis by comparing sets of data.

D E The student collected data to test a hypothesis.

F G The student organized the data.

M4 b Statistics and Probability Concepts: The student displays data in…graphs, tables, and charts.

F G

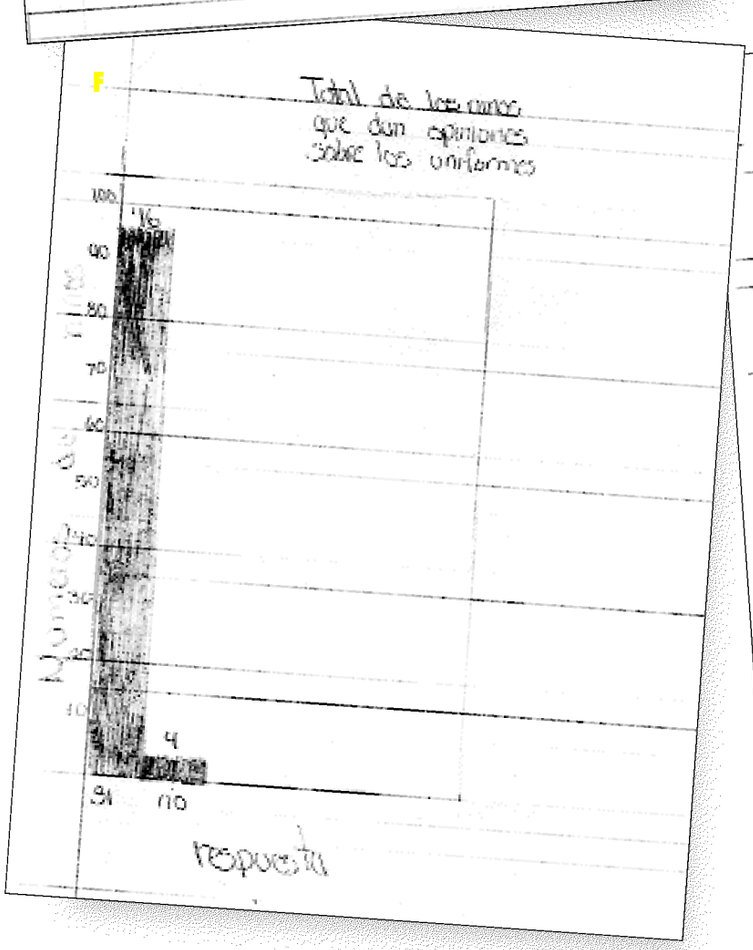
M4 c Statistics and Probability Concepts: The student makes statements and draws simple conclusiones based on data; that is:…

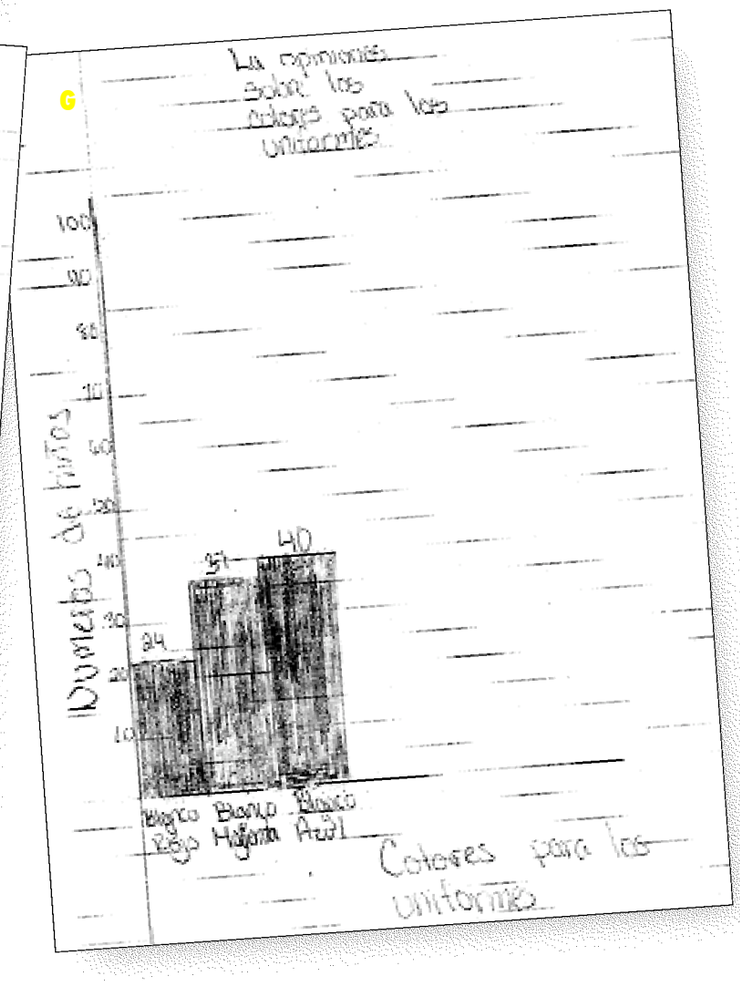
- **compares data in order to make true statements....**

C

- **uses data, including statements about the data, to make a simple concluding statement about a situation….**

H The student used the data to make simple concluding statements. Most of the statements made (e.g., "Most of the children…" for 96%) are accurate. One conclusion ("…almost all the girls…" for 25 out of 57 sampled) is not accurate.





School Uniforms Project

Arithmetic and Number Concepts M1
Geometry& Measurement Concepts
Function & Algebra Concepts
Statistics & Probability Concepts M4
Problem Solving & Reasoning M5
Mathematical Skills & Tools M6
Mathematical Communication M7
Putting Mathematics to Work M8

M4 d Statistics and Probability Concepts: The student gathers data about an entire group or by sampling group members to understand the concept of sample, i.e., that a large sample leads to more reliable information….

I The reasoning behind choosing to sample roughly 10% of the school is clearly explained.

M5 a Problem Solving and Reasoning: Formulation. Given the basic statement of a problem situation, the student.

- **makes the important decisions about the approach, materials, and strategies to use, i.e., does not merely fill in a given chart, use a pre-specified manipulative, or go through a predetermined set of steps.**

A B D E F G The student made decisions about the sampling method, and the charts and graphs to be used.

- **uses previously learned strategies, skills, knowledge, and concepts to make decisions.**

B The student used the concept of percent in making decisions.

M6 a Mathematical Skills and Tools: The student adds, subtracts, multiplies, and divides whole numbers correctly; that is:…

- **multiplies and divides three digit numbers by one digit numbers.**

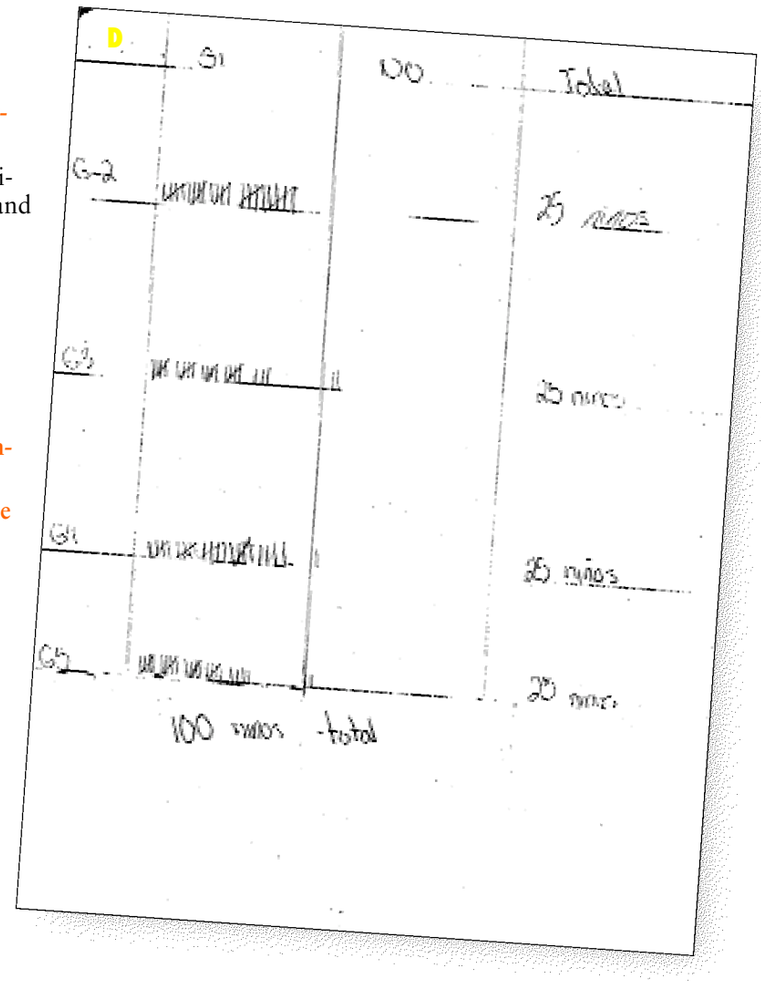
A

M6 g Mathematical Skills and Tools: The student reads, creates, and represents data on…charts, tables, diagrams, and bar graphs….

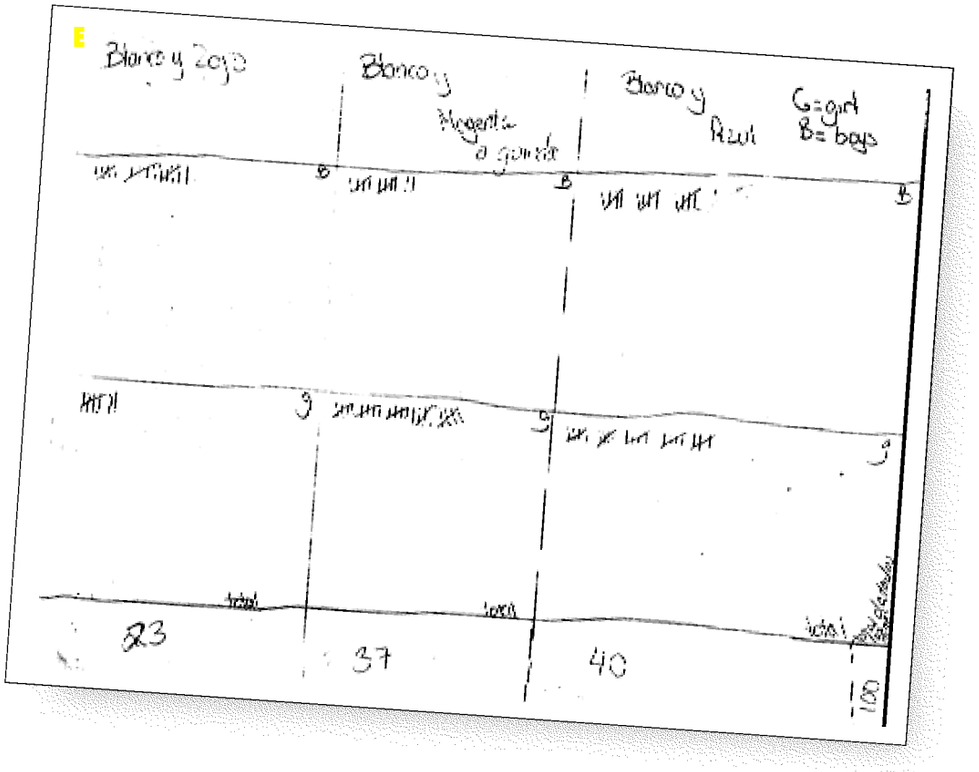
D E F G

M7 d Mathematical Communication: The student considers purpose and audience when communicating about mathematics.

J The report enables readers to understand the project.



School Uniforms Project



M8 a Putting Mathematics to Work: Data study, in which the student:

- **develops a question and a hypothesis in a situation where data could help make a decision or recommendation.**

J K The student showed clearly that the question could lead directly to a real recommendation.

The project does not entirely fulfill this criterion for the data study, because the question is not expressed as a hypothesis. Nevertheless, the student accomplished the important purposes of a large scale mathematics project. The purpose of the Putting Mathematics to Work standard is for students to make important decisions about using mathematics and to put extensive mathematics to work in realistic situations.

- **decides on a group or groups to be sampled and makes predictions of the results, with specific percents, fractions, or numbers.**

B I L The student used clear arithmetic and logical reasons in choosing the group and numbers used in the sample. The prediction applied to only one of the two questions posed. The prediction included no specific numbers. This lack of specificity does not of itself restrict the project from meeting the standard for a data study.

- **collects, represents, and displays data in order to help make the decision or recommendation; compares the results with the predictions.**

F G H

- **writes a report that includes recommendations supported by diagrams, charts, and graphs, and acknowledges assistance received from parents, peers, and teachers.**

A B C D E F G H

I J K L

School Uniforms Project

Arithmetic and Number Concepts M1
Geometry& Measurement Concepts
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Putting Mathematics to Work M8

Translation

Would you like to have uniforms at this school?

What colors would you like the uniforms to be?

J white and red
white and magenta
white and blue

Diana, Ivan, Caroline and I are going to ask the children of the school if they would like uniforms. If they answer yes, we are going to ask them what colors they would like the uniforms to be.

We think that it is important to know this because there are less problems and the children will respect one another.

The principal and the vice principal would like to know this because they would order them and because the children would wear them.

B We have 950 children in the school rounded off to 1,000 and we are going to ask 10% or 100 children in the school from kindergarten to fifth.

At first we thought we were going to ask all the grades but we changed our minds because it was hard for us to be asking questions of all the kids in the school and besides the children in kindergarten and first have a

I little trouble answering the questions. That's why we decided to ask only grades 2-5 and 25 children at each grade. But first we need to find out how many rooms there are at each grade, so we can ask the same quantity of children at each grade.

First we did some mathematics we divided 100 by 6 but there was a remainder of 1. Next we divided 100 by 5 and the result was 20 but the

A children in first grade have a hard time answering the question. Finally we found the solution we divided 100 by 4 25 and we are going to use this fraction for our survey.

I think that most of the boys are going to choose blue and white and the

L girls are going to choose red and white. But I would like it if they chose magenta and white.

Most of the children said that they would like to have uniforms only 4 children said that they would not like to have uniforms. Many boys and

H girls chose the colors blue and white, I guessed part of it but almost all the girls chose blue and white.

Many children chose that they would like uniforms $\frac{96}{100}$ 96 out of 100

C children said they wanted uniforms. $\frac{4}{100}$ 4 out of 100 children said they did not want uniforms.

$\frac{23}{96}$ 23 out of 96 children chose the colors red and white

$\frac{37}{96}$ 37 out of 96 children chose the colors magenta or guinda with white.
< Note: guinda is just another word for magenta.
(by teacher)

$\frac{40}{96}$ 40 out of 96 children chose the colors blue and white.

K I recommend to the principal and the vice-principal that they order blue and white uniforms.

I put 96 for these because because 4 children said that they didn't want uniforms and I subtracted 100 4 and the answer was 96 with the children that said they did not want uniforms we did not ask them what colors they would like.

Graph titled: Total of children who give opinions about uniforms

Graph titled. Opinions about the colors of the uniforms

tally sheet for the surveying of the children by grade

tally sheet for the surveying of the children separated by boy/girl

Work Sample & Commentary: *Catapult Investigation*

The task

After each student had built an individual catapult according to the teacher's design, the teacher gave the following instructions:

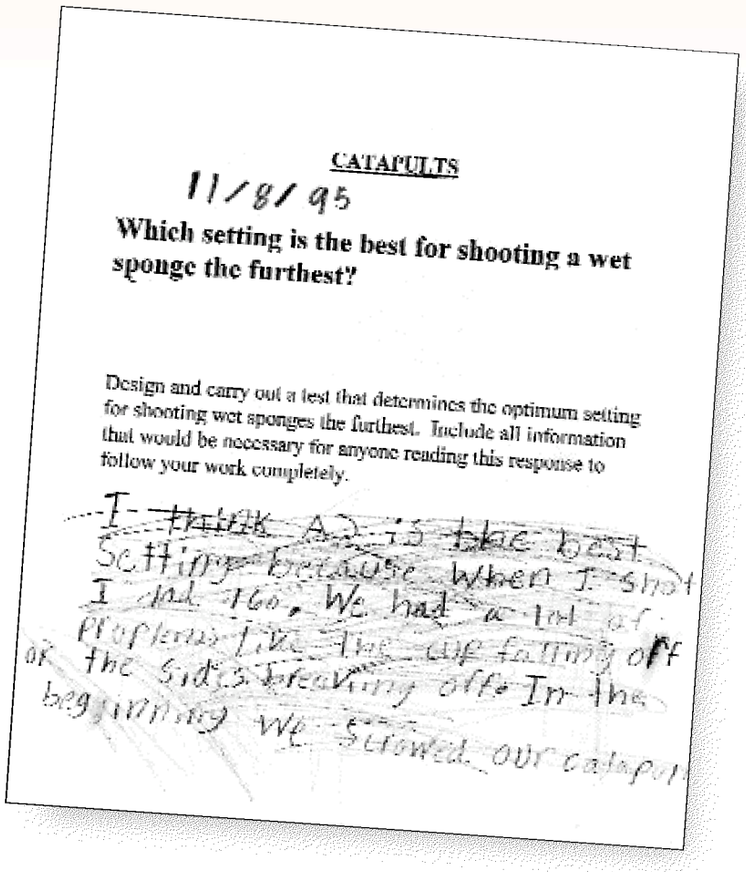
Use your catapults to determine which setting (on the catapult) is best for shooting a wet sponge the furthest. To complete the project you will need to:

- design and carry out a test that determines the optimum setting for shooting a wet sponge the furthest;
- include all information that would be necessary for anyone reading this response to follow your work completely.

Circumstances of performance

This sample of student work was produced under the following conditions:

alone	in a group
in class	as homework
with teacher feedback	with peer feedback
timed	opportunity for revision



CATAPULTS

11/8/95

Which setting is the best for shooting a wet sponge the furthest?

Design and carry out a test that determines the optimum setting for shooting wet sponges the furthest. Include all information that would be necessary for anyone reading this response to follow your work completely.

I think A2 is the best setting because when I shot I got 160. We had a lot of problems like the cup falling off or the sides breaking off. In the beginning we screwed our catapult

This work sample illustrates a standard-setting performance for the following parts of the standards:

- **M4 a Statistics and Probability Concepts: Collect and organize data to answer a question.**
- **M4 b Statistics and Probability Concepts: Display data.**
- **M4 c Statistics and Probability Concepts: Make statements and draw simple conclusions based on data.**
- **M4 f Statistics and Probability Concepts: Find all possible combinations.**
- **M5 a Problem Solving and Reasoning: Formulation.**
- **M5 b Problem Solving and Reasoning: Implementation.**
- **M6 c Mathematical Skills and Tools: Measure accurately in both the customary and metric systems.**
- **M6 g Mathematical Skills and Tools: Read, create, and represent data.**
- **M6 h Mathematical Skills and Tools: Use measuring devices and calculators to achieve solutions.**
- **M7 b Mathematical Communication: Show mathematical ideas in a variety of ways.**
- **M8 b Putting Mathematics to Work: Science study.**

Students completed this project as part of a unit on simple machines and had explored levers and fulcrums. The class took several days to complete the testing of catapult-setting variables and to write up the results as a report. Students worked with partners in the testing of their catapults.

What the work shows

M4 a Statistics and Probability Concepts: The student collects and organizes data to answer a question…by comparing sets of data.

A B The student collected, organized, and compared data in order to determine which catapult setting sends wet sponges the furthest.

M4 b Statistics and Probability Concepts: The student displays data in line plots, graphs, tables, and charts.

A Data are displayed in a clear table.

B Data are displayed in a graph. The data for settings A1 and A2 were placed off line but this does not detract from the overall clarity of the graph.

M4 c Statistics and Probability Concepts: The student makes statements and draws simple conclusions based on data; that is:…

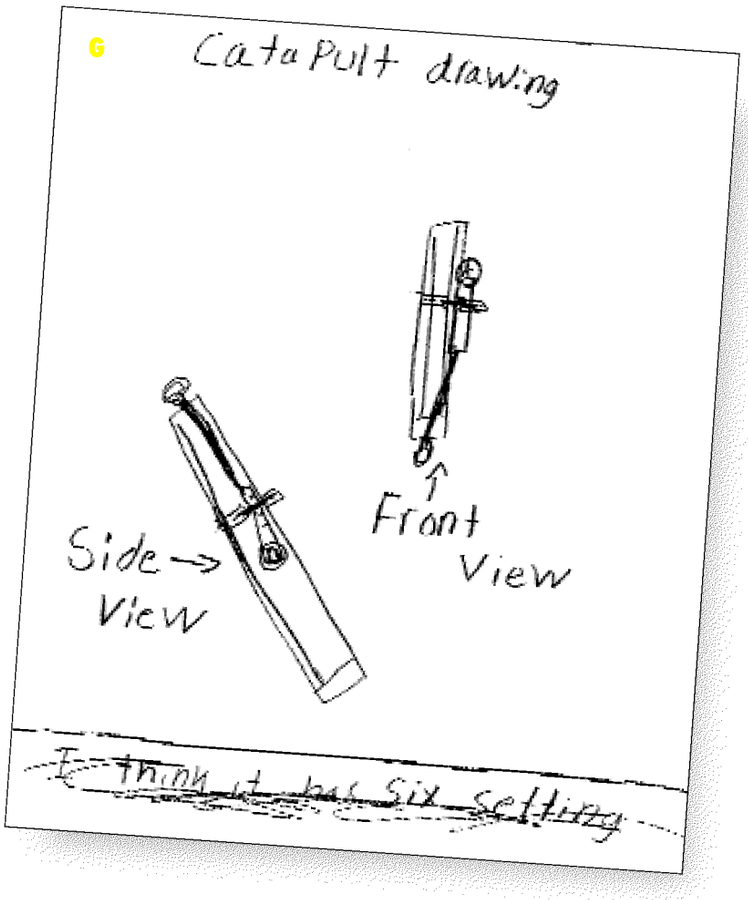
- **compares data in order to make true statements….**

C The student compared the data to make true statements: "A2 was the best setting because (it) shot very far…It had an average of 123 inches…C2 is the worst setting…."

The quotations from the Mathematics performance descriptions in this commentary are excerpted. The complete performance descriptions are shown on pages 60-65.

Catapult Investigation

Arithmetic and Number Concepts
Geometry& Measurement Concepts
Function & Algebra Concepts
Statistics & Probability Concepts M4
Problem Solving & Reasoning M5
Mathematical Skills & Tools M6
Mathematical Communication M7
Putting Mathematics to Work M8

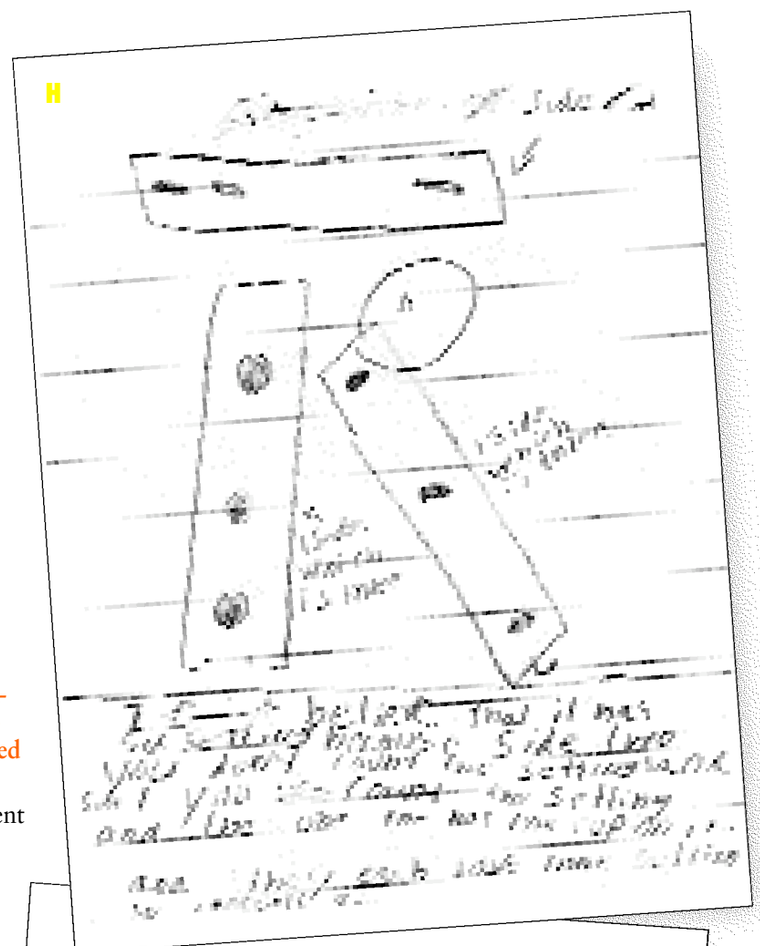


- uses previously learned strategies, skills, knowledge, and concepts to make decisions.

C The student used the previously learned concept of average to decide on the best setting.

- uses strategies, such as using manipulatives or drawing sketches, to model problems.

G H I



- makes true statements based on a simple concept of average (median and mean), for a small sample size....

A C

- uses data, including statements about the data, to make a simple concluding statement about a situation....

D

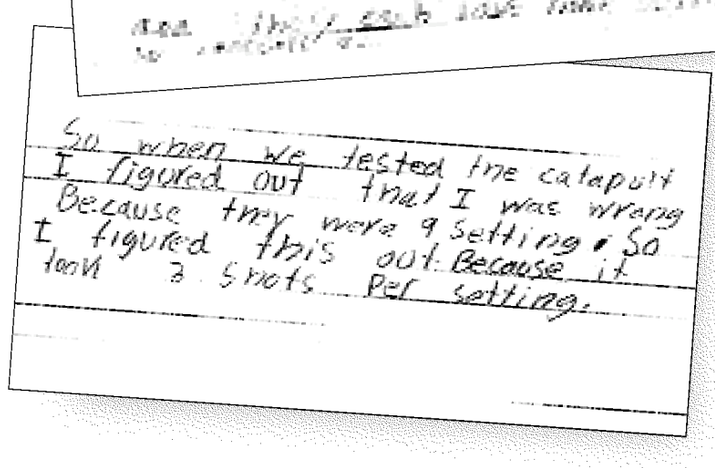
M4 f Statistics and Probability Concepts: The student finds all possible combinations and arrangements within certain constraints involving a limited number of variables.

E In the course of testing the settings, the student found all the possible combinations of settings.

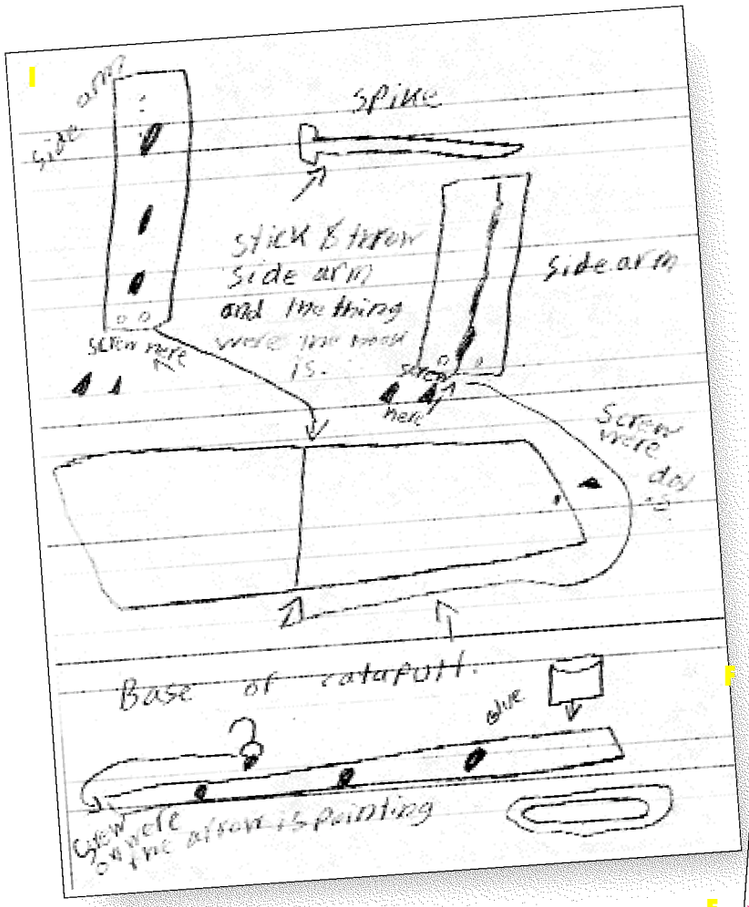
M5 a Problem Solving and Reasoning: Formulation. Given the basic statement of a problem situation, the student:

- makes the important decisions about the approach, materials, and strategies to use, i.e., does not merely fill in a given chart, use a prespecified manipulative, or go through a predetermined set of steps.

A B C E F The student decided on the methods for determining, recording, and representing the most effective setting.



Catapult Investigation



M6 g Mathematical Skills and Tools: The student reads, creates, and represents data on…charts, tables, diagrams….

A B

F In the beginning we scrawed our Catapults together. Then we talked about the proplem there would be and whow we would solve them. One day we went out side to test and to prove How many setting there were. When we fired them catapults figured out they were nine setting So I was wrong*

E They were nine setting because you have to fire like A1 A2 A3 there's three settings then C1 C2 C3 and there's nine setting when you add all togther. We solved most of the proplems but there more proplems. We decided that A1 was at the top and B was in middle and C was at bottom. There is ~~by~~ the two is in the middle and 1 is at the end. We averaged because we had a vote and it was the ~~all~~ fairest way. I thought A2 was the best setting because shot very far. I think I had an average of 123 inches!

*because I thought there were 6

M5 b Problem Solving and Reasoning: Implementation. The student makes the basic choices involved in planning and carrying out a solution; that is, the student:

- **makes up and uses a variety of strategies and approaches to solving problems and uses or learns approaches that other people use, as appropriate.**

A B C E F The student created an approach for testing the settings.

- **makes connections among concepts in order to solve problems.**

A E The student made connections among statistics concepts (e.g., finding all combinations of settings and averaging like results) to answer the question.

- **solves problems in ways that make sense and explains why these ways make sense, e.g., defends the reasoning, explains the solution.**

C D E F

C

D A2 shoots far because you have the most leverage. C2 is the worst setting because you can't lift the lever high.

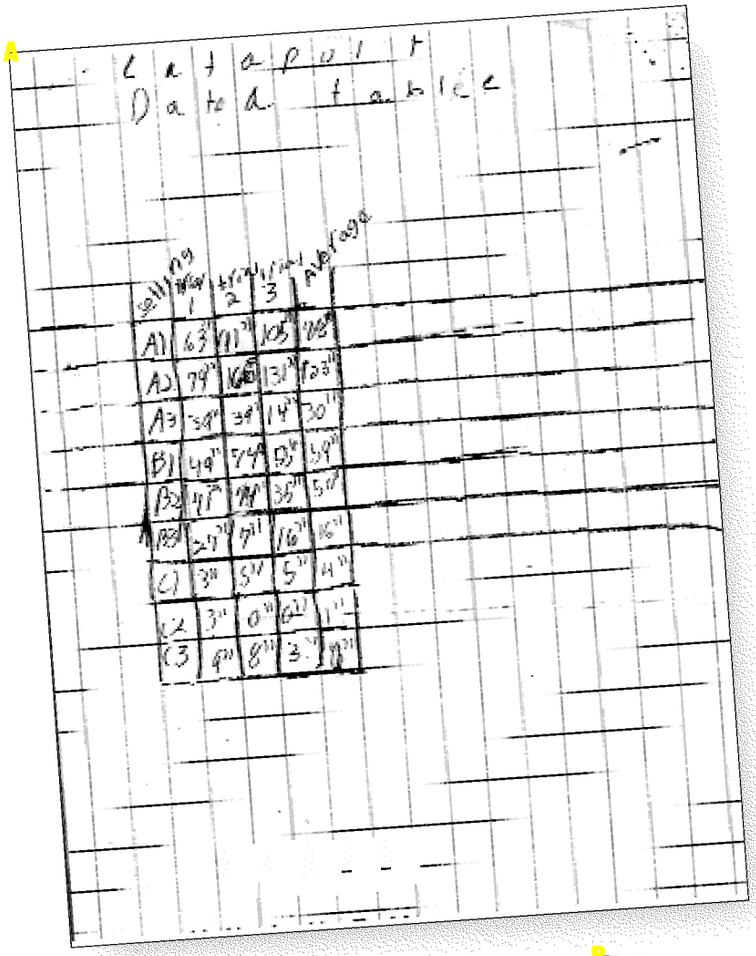
M6 c Mathematical Skills and Tools: The student measures length…accurately in both the customary and metric systems.

A The student measured the lengths of firings. The teacher verified the accuracy of measurements.

Catapult Investigation

Arithmetic and Number Concepts
Geometry & Measurement Concepts
Function & Algebra Concepts
Statistics & Probability Concepts M4
Problem Solving & Reasoning M5
Mathematical Skills & Tools M6
Mathematical Communication M7
Putting Mathematics to Work M8

A



M6 h Mathematical Skills and Tools: The student uses…measuring devices…calculators,…as appropriate, to achieve solutions.

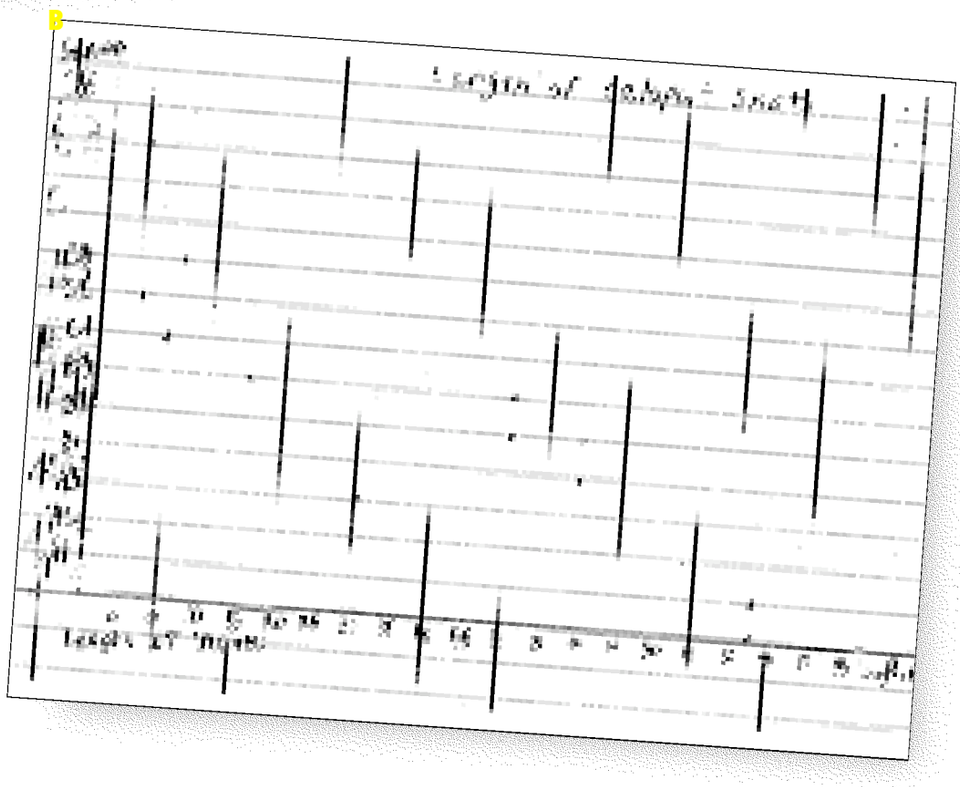
A The student used measuring devices to achieve a solution.

A The student used a calculator (as verified by the teacher) to compute averages.

M7 b Mathematical Communication: The student shows mathematical ideas in a variety of ways, including words, numbers, symbols, pictures, charts, graphs, tables, diagrams, and models.

A B F G H I The student showed his ideas in multiple ways. Some of the diagrams in particular are unlabeled, labeled confusingly, or difficult to interpret (see the three diagrams of the catapult). Overall, however, the ideas are communicated with sufficient clarity.

B



Catapult Investigation

M8 b Putting Mathematics to Work: The student conducts a science study, in which the student:

- **decides on a specific science question to study and identifies the mathematics that will be used, e.g., measurement.**

A B C E F The student investigated a science question and decided what mathematics should be used.

- **develops a prediction (a hypothesis) and develops procedures to test the hypothesis.**

E F G H I This investigation does not entirely fit the criteria for a "Science Study," because it does not ask the student for a hypothesis. The purpose of the Putting Mathematics to Work standard is for students to put extensive mathematics to work in realistic situations. The investigation clearly accomplishes this central purpose of a large scale mathematics project.

- **collects and records data, represents and displays data, and compares results with predictions.**

A B C The data are recorded, represented, and compared. Since the teacher called for students to answer a question in this investigation, the results of the data collection are used to this end, rather than to make a prediction.

- **writes a report that compares the results with the hypothesis; supports the results with diagrams, charts, and graphs; acknowledges assistance received from parents, peers, and teachers.**

The report is clear and uses the results to answer the question. The teacher did not ask students to acknowledge assistance; nevertheless, the student acknowledged his partner's assistance in making certain decisions.

Work Sample & Commentary: *The Never Ending Four*

Arithmetic and Number Concepts
Geometry & Measurement Concepts
Function & Algebra Concepts M3
Statistics & Probability Concepts M4
Problem Solving & Reasoning M5
Mathematical Skills & Tools M6
Mathematical Communication M7
Putting Mathematics to Work M8

The quotations from the Mathematics performance descriptions in this commentary are excerpted. The complete performance descriptions are shown on pages 60-65.

The task

The teacher asked students to complete a large scale mathematics project chosen from among five kinds. This student chose to do a "Pure Mathematics Investigation." The teacher gave the student the written instructions illustrated here.

Circumstances of performance

This sample of student work was produced under the following conditions:

alone	in a group
in class	as homework
with teacher feedback	with peer feedback
timed	opportunity for revision

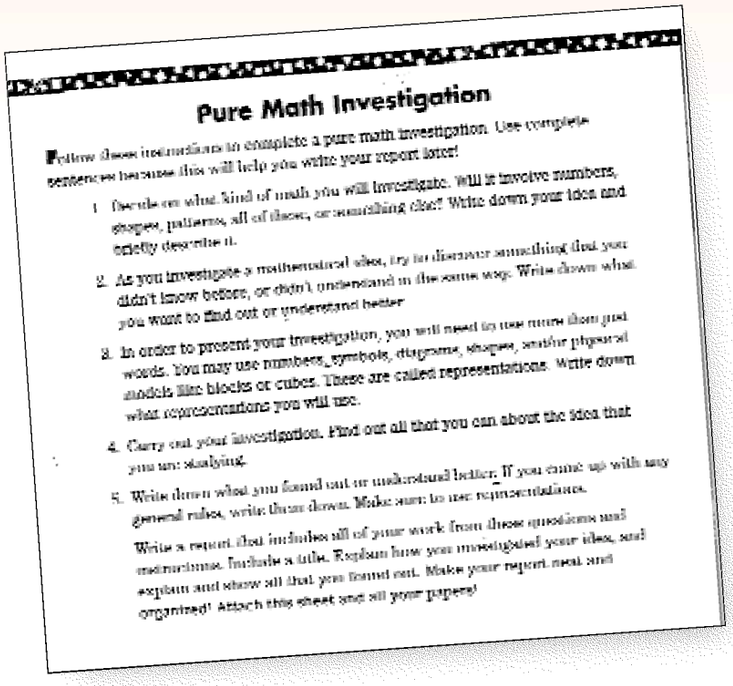
In this classroom students were using the New Standards Elementary Mathematics Portfolio. The teacher intended students to be able to use their chosen projects to fulfill the requirements for the project exhibit in the portfolio.

What the work shows

M3 b Function and Algebra Concepts: The student builds iterations of simple non-linear patterns, including multiplicative and squaring patterns (e.g., "growing" patterns) with concrete materials, and recognizes that these patterns are not linear.

This work sample illustrates a standard-setting performance for the following parts of the standards:

- **M3 b Function and Algebra Concepts: Build iterations of simple non-linear patterns and recognize that these patterns are not linear.**
- **M4 a Statistics and Probability Concepts: Collect and organize data to answer a question.**
- **M5 a Problem Solving and Reasoning: Formulation.**
- **M6 g Mathematical Skills and Tools: Read, create, and represent data.**
- **M7 b Mathematical Communication: Show mathematical ideas in a variety of ways.**
- **M7 c Mathematical Communication: Explain solutions to problems clearly and logically.**
- **M7 d Mathematical Communication: Consider purpose and audience when communicating about mathematics.**
- **M8 e Putting Mathematics to Work: Pure mathematics investigation.**



Pure Math Investigation

Follow these instructions to complete a pure math investigation. Use complete sentences because this will help you write your report later!

1. Decide on what kind of math you will investigate. Will it involve numbers, shapes, patterns, all of these, or something else? Write down your idea and briefly describe it.
2. As you investigate a mathematical idea, try to discover something that you didn't know before, or didn't understand in the same way. Write down what you want to find out or understand better.
3. In order to present your investigation, you will need to use more than just words. You may use numbers, symbols, diagrams, shapes, and/or physical models like blocks or cubes. These are called representations. Write down what representations you will use.
4. Carry out your investigation. Find out all that you can about the idea that you are studying.
5. Write down what you found out or understand better. If you came up with any general rules, write them down. Make sure to use representations.

Write a report that includes all of your work from these questions and instructions. Include a title. Explain how you investigated your idea, and explain and show all that you found out. Make your report neat and organized! Attach this sheet and all your papers!

A The student built (again and again) the non-linear pattern generated by this "trick" question (i.e., that the numbers appearing before a four turns up are always five or nine).

M4 a Statistics and Probability Concepts: The student collects and organizes data to answer a question…by comparing sets of data.

B The student represented data in a table in order to further explore the number relationships.

M5 a Problem Solving and Reasoning: Formulation. Given the basic statement of a problem situation, the student:

- **makes the important decisions about the approach, materials, and strategies to use, i.e., does not merely fill in a given chart, use a pre-specified manipulative, or go through a predetermined set of steps.**

A B C D E The student made all decisions regarding how to proceed with this entirely unformulated project.

M6 g Mathematical Skills and Tools: The student reads, creates, and represents data on charts, tables, diagrams….

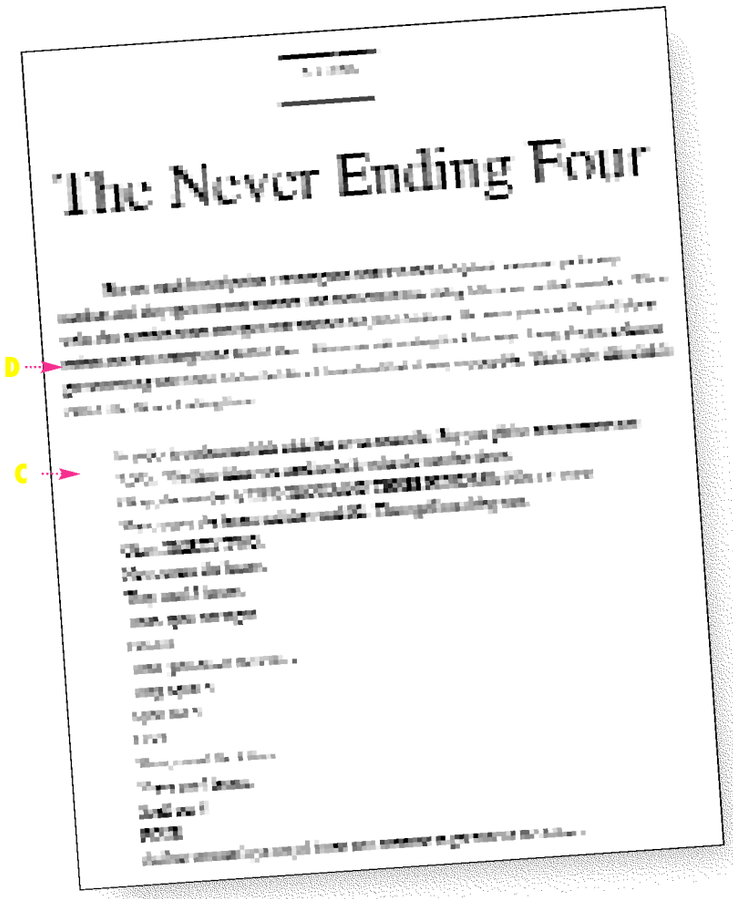
B

M7 b Mathematical Communication: The student shows mathematical ideas in a variety of ways, including words, numbers, symbols,…charts, tables, diagrams….

A B C D E

M7 c Mathematical Communication: The student explains solutions to problems clearly and logically, and supports solutions with evidence, in both oral and written work.

The Never Ending Four



B C The student used a narrative description and tables to illustrate the nature of the "problem" (or, in this case, "trick").

M7 d Mathematical Communication: The student considers purpose and audience when communicating about mathematics.

D E The student clearly explained the nature of the project as well as how it would be conveyed to the classroom audience.

M8 e Putting Mathematics to Work: The student conducts a pure mathematics investigation, in which the student:

- **decides on the area of mathematics to investigate, e.g., numbers, shapes, patterns.**

  D

- **describes a question or concept to investigate.**

  D

- **decides on representations that will be used, e.g., numbers, symbols, diagrams, shapes, or physical models.**

  A B C D E

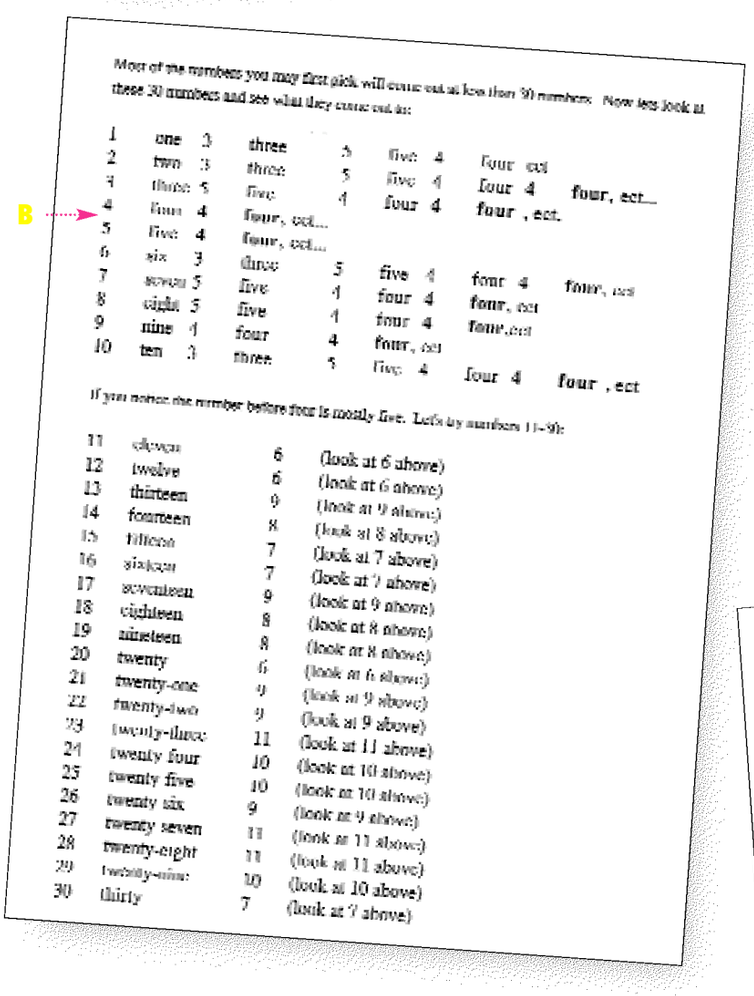
- **carries out the investigation.**

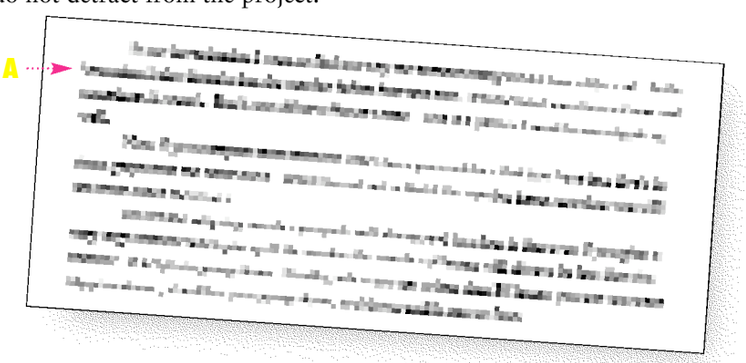
  A B C D E

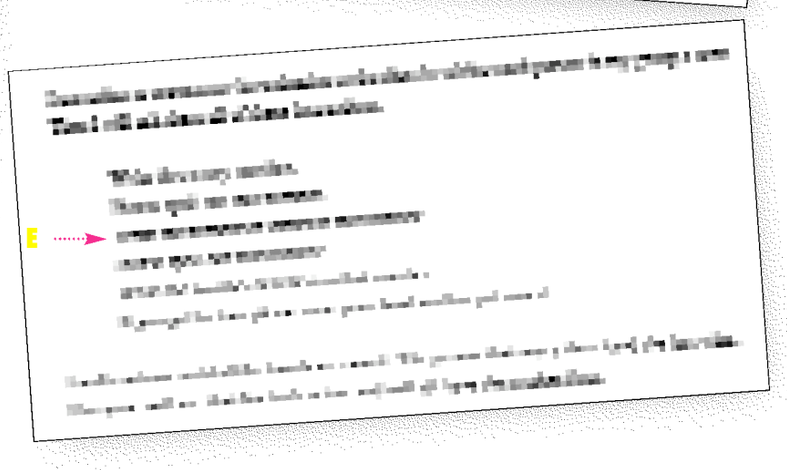
- **writes a report that includes any generalizations drawn from the investigation, and acknowledges assistance received from parents, peers, and teachers.**

  A B C D E

Three errors in counting the letters (32, 13, and 30) do not detract from the project.







Work Sample & Commentary: *Dream House Project*

Arithmetic and Number Concepts M1
Geometry & Measurement Concepts M2
Function & Algebra Concepts
Statistics & Probability Concepts M4
Problem Solving & Reasoning
Mathematical Skills & Tools M6
Mathematical Communication M7
Putting Mathematics to Work M8

The quotations from the Mathematics performance descriptions in this commentary are excerpted. The complete performance descriptions are shown on pages 60-65.

The task

The teacher gave students the following instructions, and clarified them through an extended discussion:

Plan and design your own "dream house" within certain cost constraints. Here are the budget, cost, and building rules you will have to work with.

- You will have $200,000 to spend altogether.
- The land for the house costs $100,000.
- Construction of "regular" rooms (traditional rooms with no special requirements) costs $75 per square foot.
- Construction of "special" rooms (requiring special wiring, plumbing, or unusual materials) costs $150 per square foot.
- All houses must include a kitchen, bathroom, bedroom, and living room.
- Rooms and hallways must have reasonable areas.
- The overall design must be convenient and practical (e.g., provide easy access to rooms, allow for privacy, include doors and hallways in practical locations, etc.).
- The number of sides to your floor plan should be limited, to avoid a sprawling, awkward design.

To get started, you will:

- Complete a rough draft of your floor plan by cutting out the rooms from half-inch graph paper.
- Record the calculations of the area and cost on the cut-out of the room.
- Track your budget on a running budget sheet ("Budget Update").
- Meet in a response group to see if your rough draft is reasonable.

To finish, you will:

- Make a final draft of your floor plan without graph paper, using a ruler and protractor, and following the scale exactly.
- Draw a front and side view of your dream house based on your floor plan.

This work sample illustrates a standard-setting performance for the following parts of the standards:

- M1 a **Arithmetic and Number Concepts: Add, subtract, multiply, and divide whole numbers.**
- M2 b **Geometry and Measurement Concepts: Visualize and represent two dimensional views of simple rectangular three dimensional shapes.**
- M2 g **Geometry and Measurement Concepts: Use basic ways of estimating and measuring the size of figures and objects in the real world.**
- M2 k **Geometry and Measurement Concepts: Use scales in maps and scale drawings.**
- M4 c **Statistics and Probability Concepts: Make statements and draw simple conclusions based on data.**
- M6 a **Mathematical Skills and Tools: Add, subtract, multiply, and divide whole numbers correctly.**
- M6 c **Mathematical Skills and Tools: Measure length, area, and perimeter accurately.**
- M6 h **Mathematical Skills and Tools: Use measuring devices.**
- M7 a **Mathematical Communication: Use appropriate mathematical terms, vocabulary, and language.**
- M7 b **Mathematical Communication: Show mathematical ideas in a variety of ways.**
- M7 d **Mathematical Communication: Consider purpose and audience when communicating about mathematics.**
- M8 c **Putting Mathematics to Work: Design a physical structure.**

Circumstances of performance

This sample of student work was produced under the following conditions:

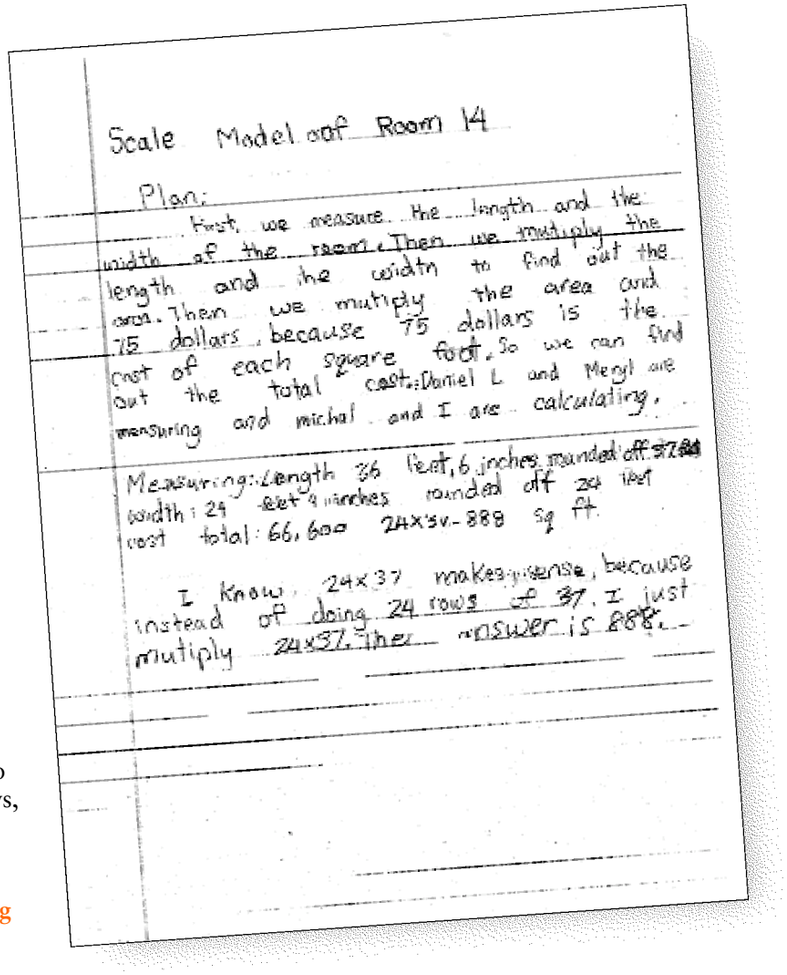
alone	in a group
in class	as homework
with teacher feedback	with peer feedback
timed	opportunity for revision

Students completed this project over the course of four weeks. They worked individually except when initially measuring the classroom as part of a pre-project activity ("Scale Model of Room 14"). They also met once in small peer response groups, and once with an editing partner.

The teacher reviewed students' progress on this project daily, and created the "House Project Response Group Meeting Notes" and the "Editing Checklist for House Description" in order to help students complete the project.

The students had worked with two digit multiplication and the concept of area prior to beginning the project. They were allowed to use calculators to check calculations and for multiplication with multipliers of three or more digits.

Dream House Project



Scale Model of Room 14

Plan:
First we measure the length and the width of the room. Then we mutiply the length and the width to find out the area. Then we mutiply the area and 75 dollars, because 75 dollars is the cost of each square foot. So we can find out the total cost. Daniel L and Megl are measuring and michal and I are calculating.

Measuring: Length 36 feet, 6 inches rounded off 37 feet
width: 24 feet 4 inches rounded off 24 feet
cost total: 66,600 24x37=888 sq ft.

I know 24x37 makes sense, because instead of doing 24 rows of 37, I just mutiply 24x37. ther answer is 888.

What the work shows

M1 a Arithmetic and Number Concepts: The student adds, subtracts, multiplies…whole numbers with and without calculators; that is:

- **adds, i.e., joins things together, increases.**

A The student added costs of rooms to determine the amount remaining in the budget.

- **subtracts, i.e., takes away, compares, finds the difference.**

B The student subtracted to determine the amount of the budget remaining.

- **multiplies, i.e., uses repeated addition, counts by multiples….**

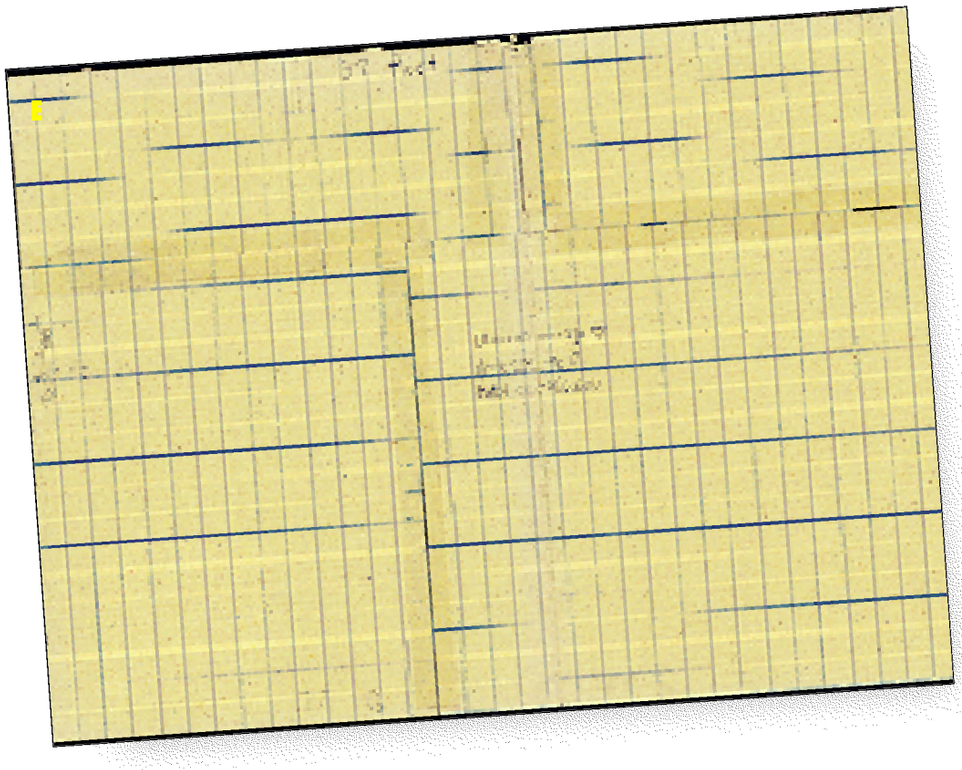
C The student multiplied the area of rooms by cost per square foot to determine the total cost of each room.

M2 b Geometry and Measurement Concepts: The student visualizes and represents two dimensional views of simple rectangular three dimensional shapes….

D The student visualized and represented the two dimensional house floor plan in side and front views, i.e., as if in three dimensions.

M2 g Geometry and Measurement Concepts: The student uses basic ways of estimating and measuring the size of figures and objects in the real world, including length, width, perimeter, and area.

E The student measured the length and width of the classroom in order to determine area.



Dream House Project

Arithmetic and Number Concepts M1
Geometry & Measurement Concepts M2
Function & Algebra Concepts
Statistics & Probability Concepts M4
Problem Solving & Reasoning
Mathematical Skills & Tools M6
Mathematical Communication M7
Putting Mathematics to Work M8

M2 k Geometry and Measurement Concepts: The student uses…scales…for rectangular scale drawings based on work with concrete models and graph paper.

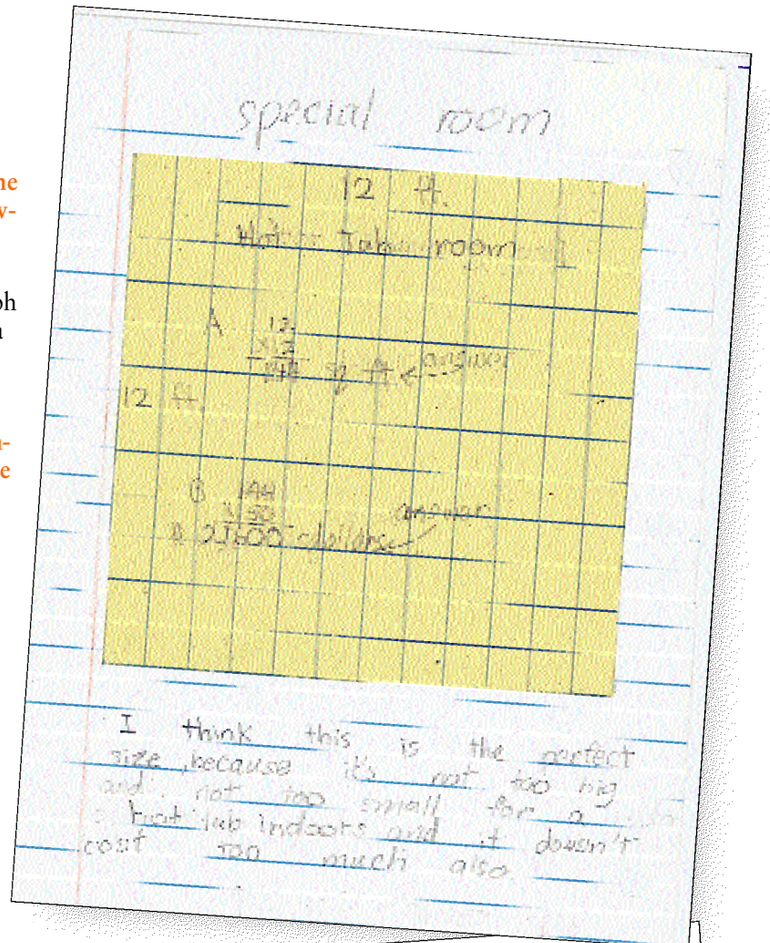
E C The student used half-inch square graph paper to create a ½ inch: 1 foot scale model of a room.

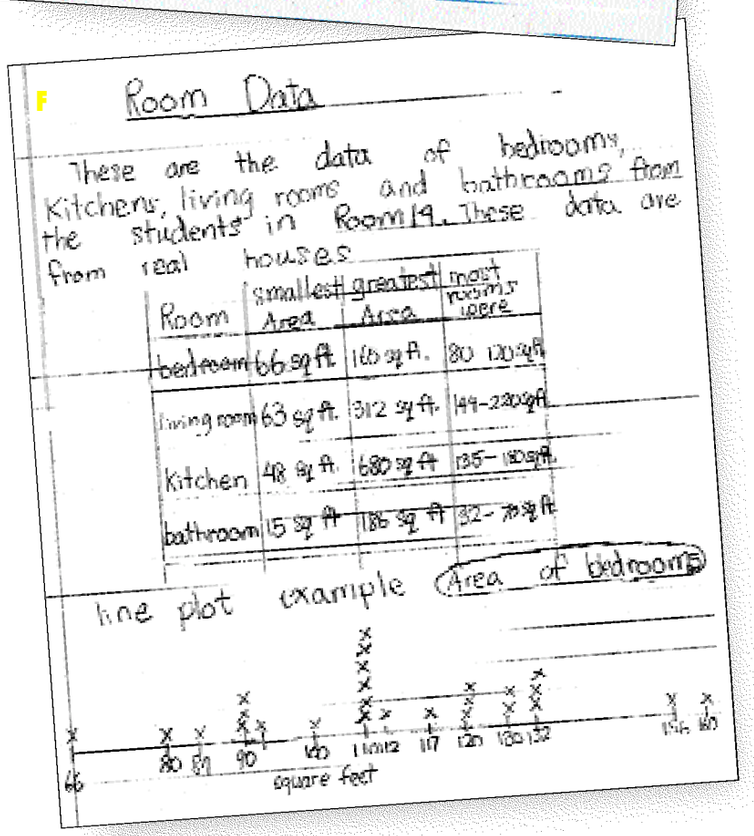
M4 c Statistics and Probability Concepts: The student makes statements and draws simple conclusions based on data; that is, reads data in line plots….

F The student interpreted line plot data in order to make decisions about reasonable areas of rooms in the floor plan.

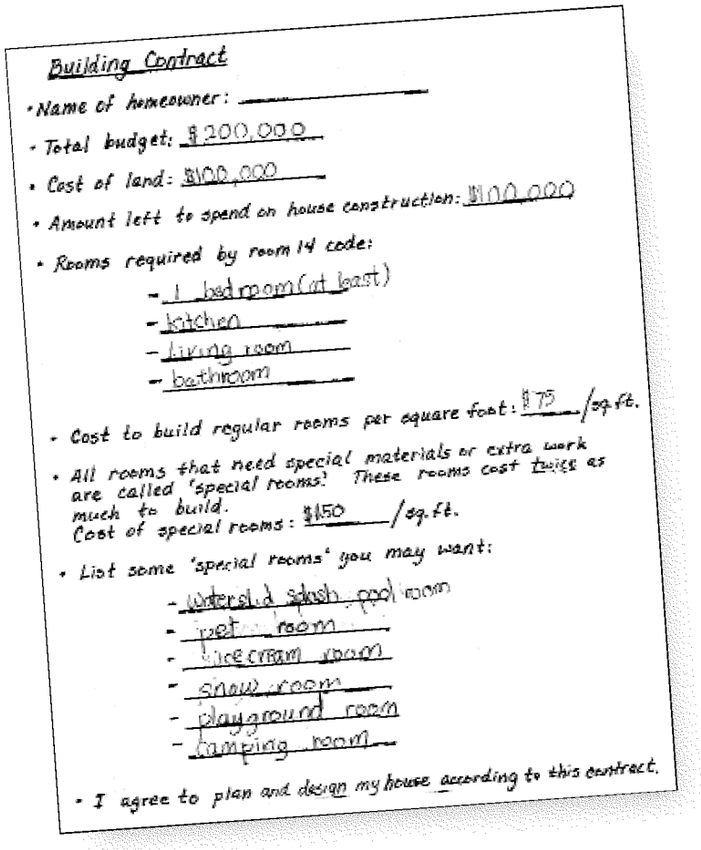
M6 a Mathematical Skills and Tools: The student adds, subtracts, multiplies…whole numbers correctly.

A B C There is evidence of accurate calculations in these parts of the work and throughout.





Dream House Project



Building Contract

- Name of homeowner: _____
- Total budget: $200,000
- Cost of land: $100,000
- Amount left to spend on house construction: $100,000
- Rooms required by room 14 code:
  - 1 bedroom (at least)
  - kitchen
  - living room
  - bathroom
- Cost to build regular rooms per square foot: $75 /sq.ft.
- All rooms that need special materials or extra work are called 'special rooms.' These rooms cost twice as much to build. Cost of special rooms: $150 /sq.ft.
- List some 'special rooms' you may want:
  - water slide splash pool room
  - pet room
  - ice cream room
  - snow room
  - playground room
  - camping room
- I agree to plan and design my house according to this contract.

M6 c Mathematical Skills and Tools: The student measures length, area,…perimeter…accurately in…the customary…system.

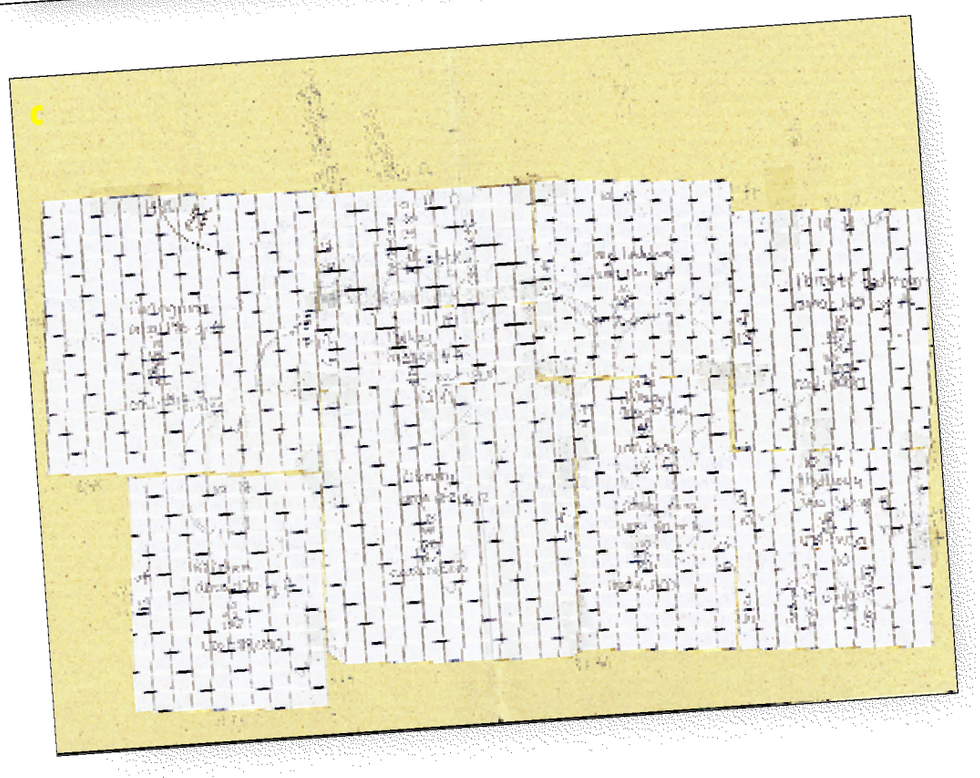
C E These parts of the work provide evidence of accurate calculations that are supported by the calculations shown in the rough and final drafts.

M6 h Mathematical Skills and Tools: The student…uses measuring devices graded appropriately for given situations, such as rulers (customary to the ⅛ inch…), graph paper (customary to the inch or half-inch…)….

C E G The student used a ruler and graph paper to measure lengths of sides of rooms.

M7 a Mathematical Communication: The student uses appropriate mathematical terms, vocabulary, and language, based on prior conceptual work.

H The student used appropriate mathematical vocabulary to describe the process involved in completing the project, e.g., "area," "unit," "reasonable."



Dream House Project

Arithmetic and Number Concepts M1
Geometry & Measurement Concepts M2
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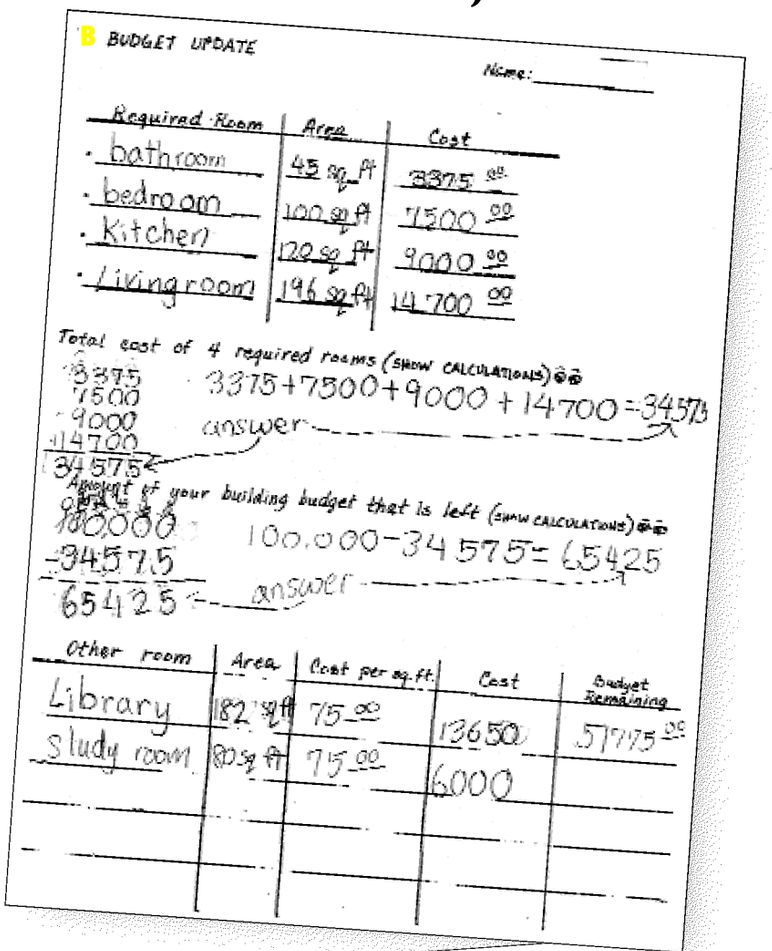


B BUDGET UPDATE

Name: _____

Required Room	Area	Cost
bathroom	45 sq ft	3375 00
bedroom	100 sq ft	7500 00
Kitchen	120 sq ft	9000 00
Living room	196 sq ft	14700 00

Total cost of 4 required rooms (SHOW CALCULATIONS)

3375
7500
9000
+14700
34575

3375+7500+9000+14700=34575

answer

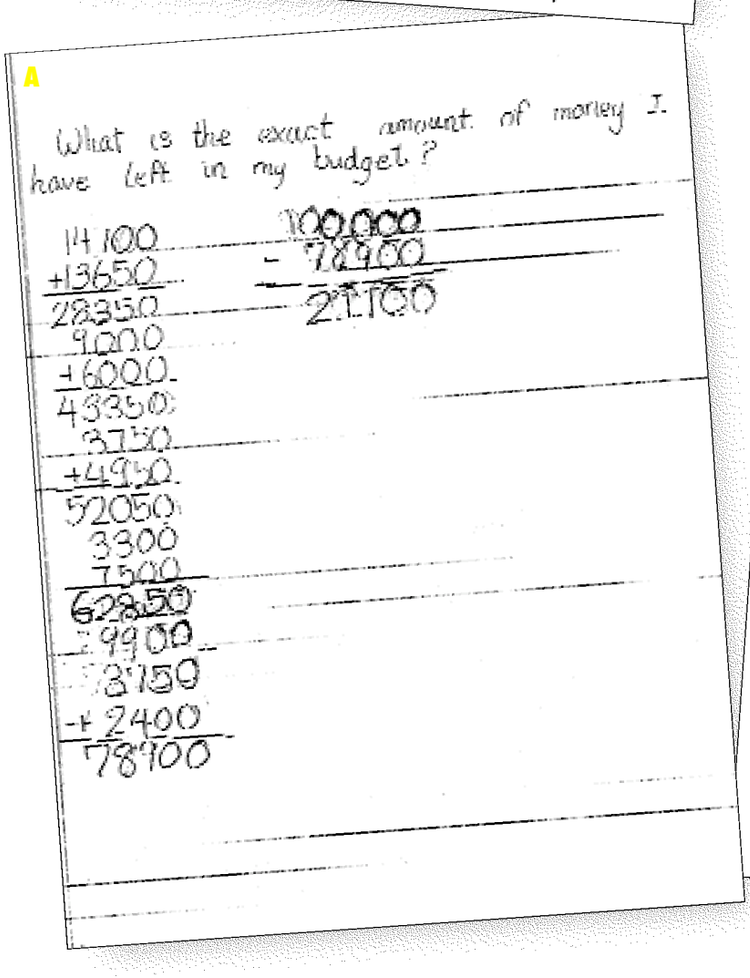
Amount of your building budget that is left (SHOW CALCULATIONS)

100,000
-34575
65425

100,000-34575=65425

answer

Other room	Area	Cost per sq ft	Cost	Budget Remaining
Library	182 sq ft	75 00	13650	51775 00
study room	80 sq ft	75 00	6000	



A

What is the exact amount of money I have left in my budget?

14100
+13650
28350
9000
+6000
43350
3750
+4950
52050
3300
7500
62850
9900
3150
+2400
78900

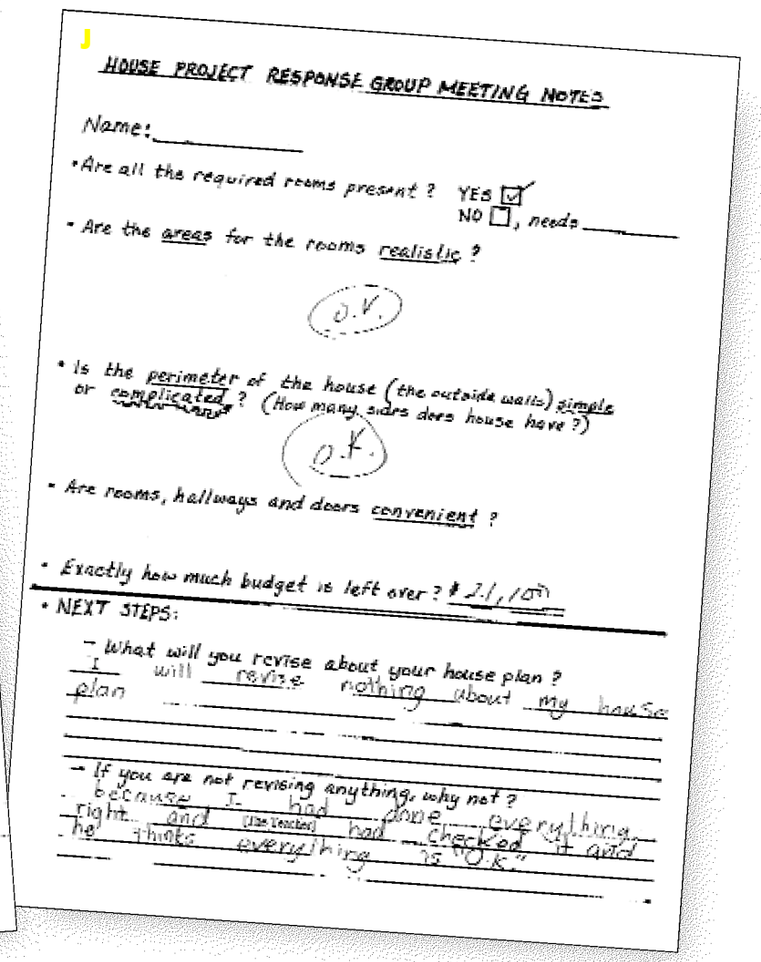
100,000
- 78900
21100

M7 b Mathematical Communication: The student shows mathematical ideas in a variety of ways, including words, numbers…graphs…tables….

B E F H

M7 d Mathematical Communication: The student considers purpose and audience when communicating about mathematics.

I The student communicated the purpose and explained the process for completing the project clearly enough for the project to be understood by an otherwise uninformed peer.



J

HOUSE PROJECT RESPONSE GROUP MEETING NOTES

Name: _____

• Are all the required rooms present? YES ☑ NO ☐, needs _____

• Are the areas for the rooms realistic?

O.K.

• Is the perimeter of the house (the outside walls) simple or complicated? (How many sides does house have?)

O.K.

• Are rooms, hallways and doors convenient?

• Exactly how much budget is left over? $ 21,100

• NEXT STEPS:

– What will you revise about your house plan?

I will revise nothing about my house plan

– If you are not revising anything, why not?

because I had done everything right and (I've checked) had checked it and he thinks everything is "O.K."

Dream House Project

House Project - REVIEW DRAFT DESCRIPTION Name: _____

Q #1: Explain the project.... What is this? What's it about? How much did you have to spend? How much does one square foot cost? What rooms did you have to have? Was the land free? What is the SCALE (how long is one foot on your plan?)

The project is a house project. It's about building our own dream house. The total we could spend for our house is 100,000.00 and the land cost 400,000.00 too (the land is not free of course) so all together including the house and the land it cost 1,500,000.00. One square foot cost 75.00 for a regular room like a bedroom or a livingroom but one square foot costs 150.00 for a special room like a swimming pool. ½ inch is 1 square foot.

Q #2: Explain about AREA.... What is area? What units do you use to measure area? Give a clear example - use one of your rooms to explain how you know how to figure out the area...

Area is how big is the whole thing. We use a square foot as one unit. My bedroom is 100 square feet, I did it by multiplying it. I count how many squares are on one side and then I count how many squares are on the top then I multiply it together and I got the answer of the whole room. I do that with the other rooms too. The answer I get of every room is how many square feet there are.

Q #3: Show how you know how to use 2-digit multiplication to calculate a room's area.... (like 16 ft. x 12 ft.). Show how to figure out how much a room costs (explain clearly... If you used a calculator, explain how.... Explain how you could still figure it out even without a calculator)

diagrams numbers pictures words

If one of my room is 13 ft x 14 ft, I first do 3x4=12. So I write the 2 and add the 10 to the 10x4 which is 40 but if I add a 10 to the 40 it's 50 so I write the 5 on the left side of 2, it's 52 Then I do 3x10 equals to 30 I write the 3 under the 5 on top from 52. Then I do 10x10=100 and I write the 1 on the left side of 3 Then I add the 52 and 130 together and I get the answer of 182 sq ft. Here's what it looks like:

```
  13
 x14
 ---
  52
 13
 ---
 182
```

To find out the cost of a regular room you multiply the total sq ft to 75.00 because each square feet of a regular room is 75.00 so I do the total square feet of the room times 75.00 and if it's a special room you also use the total square feet of that room times 150.00 because that's how much one square foot cost of a special room.

Q #4: Explain how you know that your rooms have a reasonable area (you can use the notes you already have on the back of your first draft rooms... just give a few examples - you don't need to explain about every room)

I know my rooms have a reasonable area because I looked at the rooms chart that my class had made. Those rooms are real rooms that the class had measured. Anyway, I compared my rooms to the chart the class had made and see if it is too big or too small or perfect. My rooms always turns out to be perfect and another way I could tell is by cutting out shape furnitures to put in the rooms and see if it fits.

Q #5: Explain:
- How many square feet does your finished house have?
- How much does it cost, altogether? (explain where your calculations are so they can be looked at)
- How much money is leftover in your budget? (explain how you figured this out)

My finished house have 1052 square feet because I added up every room's total square feet and I got the answer. Then I check it with the other paper when I wrote down my exact square feet of my whole house and it's the same. Then I time 1052 with 75 because that's how many square feet each square feet cost. I used the calculator. I got the answer of 78000.00. That's how much money I spend. I used the calculator. I still have 22000.00 left I used the calculator but I also have those answer in the other sheet of paper. The whole perimeter of my house is 142 ft. I did it by counting the walls of my house.

M1 Arithmetic and Number Concepts
M2 Geometry & Measurement Concepts
Function & Algebra Concepts
M4 Statistics & Probability Concepts
Problem Solving & Reasoning
M6 Mathematical Skills & Tools
M7 Mathematical Communication
M8 Putting Mathematics to Work

Dream House Project

Arithmetic and Number Concepts M1
Geometry & Measurement Concepts M2
Function & Algebra Concepts
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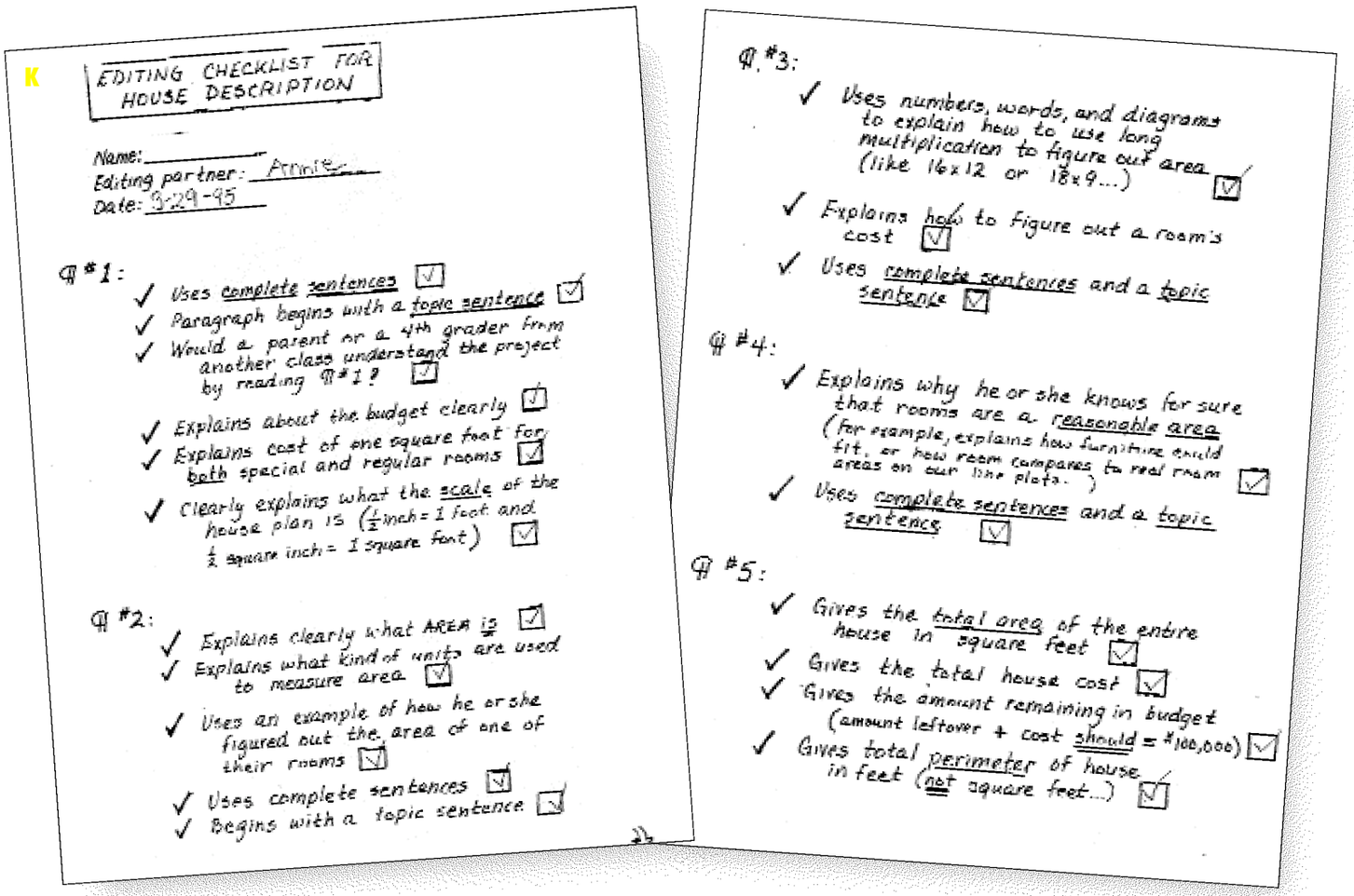


K

EDITING CHECKLIST FOR HOUSE DESCRIPTION

Name:
Editing partner: Annie
Date: 3-29-95

¶ #1:
- ✓ Uses complete sentences ☑
- ✓ Paragraph begins with a topic sentence ☑
- ✓ Would a parent or a 4th grader from another class understand the project by reading ¶ #1? ☑
- ✓ Explains about the budget clearly ☑
- ✓ Explains cost of one square foot for both special and regular rooms ☑
- ✓ Clearly explains what the scale of the house plan is ($\frac{1}{2}$ inch = 1 foot and $\frac{1}{4}$ square inch = 1 square foot) ☑

¶ #2:
- ✓ Explains clearly what AREA is ☑
- ✓ Explains what kind of units are used to measure area ☑
- ✓ Uses an example of how he or she figured out the area of one of their rooms ☑
- ✓ Uses complete sentences ☑
- ✓ Begins with a topic sentence ☑

¶ #3:
- ✓ Uses numbers, words, and diagrams to explain how to use long multiplication to figure out area (like 16x12 or 18x9...) ☑
- ✓ Explains how to figure out a room's cost ☑
- ✓ Uses complete sentences and a topic sentence ☑

¶ #4:
- ✓ Explains why he or she knows for sure that rooms are a reasonable area (for example, explains how furniture could fit, or how room compares to real room areas on our line plots.) ☑
- ✓ Uses complete sentences and a topic sentence ☑

¶ #5:
- ✓ Gives the total area of the entire house in square feet ☑
- ✓ Gives the total house cost ☑
- ✓ Gives the amount remaining in budget (amount leftover + cost should = $100,000) ☑
- ✓ Gives total perimeter of house in feet (not square feet...) ☑

M8 ‹ Putting Mathematics to Work: The student conducts a project to design a physical structure, in which the student:

- **decides on a structure to design, the size and budget constraints, and the scale of design.**

C The student made the important decisions in planning the dream house.

- **makes a first draft of the design, and revises and improves the design in response to input from peers and teachers.**

C I J K The student made revisions to the rough draft floor plan and to the rough draft description on the basis of feedback from peers and the teacher.

- **makes a final draft and report of the design, drawn and written so that another person could make the structure....**

I G

There are several spelling and grammatical errors in the rough draft work. Only the final draft description and floor plan were edited for spelling and grammar. A few errors remain in the final draft work; however, the errors do not detract from its accuracy or clarity (e.g., "mutiply" for "multiply").

Dream House Project

M1 Arithmetic and Number Concepts
M2 Geometry & Measurement Concepts
Function & Algebra Concepts
M4 Statistics & Probability Concepts
Problem Solving & Reasoning
M6 Mathematical Skills & Tools
M7 Mathematical Communication
M8 Putting Mathematics to Work

I

Designed House.

The project is a house project. It's about building our own dream house. The total we could spend for our house is $100,000 and the land cost $100,000 too (the land is not free, of course) so all together including the house and the land, it cost $200,000. One square foot cost $15.00 for a regular room like a bedroom or a livingroom, but one square foot cost $150.00 for a special room like a swimming pool. ¼ inch is one foot in the design.

H

Area is space covered in the whole thing. We use one square foot as one unit. My bedroom is 100 square feet. I did it by multiplying it. I count how many feet there are on one side and then I count how many feet there are on the top, then I multiply it together and I get the answer of how many square feet there are in the whole room. I do that with all the other rooms too.

If one of my rooms is 13 ft X 14 ft, I first do 3x4=12 so I write the 2 and add the 10 to the 4X10 which is 40 but if I add a 10 to the 40 it'll be 50 so I write the 5 on the left side of 2, it's 52. Then I do 3X10 equals to 30 I write the 3 under the 5 on top from 52. Then I do 10X10 and I write the 1 on the left side of 3. Then I add the 52 and 130 together and I got the answer of 182 square feet. Here's what it looks like:

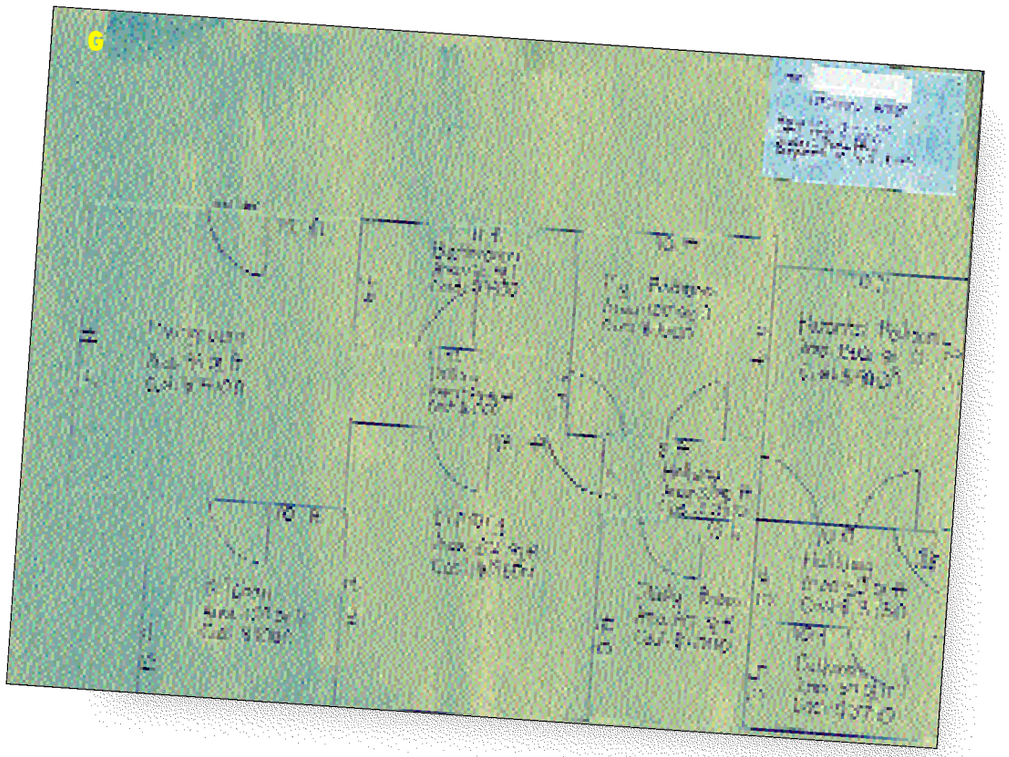
$$\begin{array}{r} 13 \\ \times 14 \\ \hline 52 \\ +13 \\ \hline 182 \end{array}$$

To find out the cost of a regular room you multiply the total square feet to $15.00, because each square foot of a regular room is $15.00 so I do the total square feet of the room times $15.00 and if the room is a special room, you then use the total square feet of that room times $150.00 because that's how much one square foot cost for a special room.

I know all my rooms have a reasonable area because I looked at the rooms chart that my class had made those rooms are real rooms that the class had measured. Anyway, I compared my rooms to the charts the class had made and see if it is too big or too small or just perfect, my rooms always turns out to be perfect. Another way I could tell is by cutting out fake furnitures to put in the rooms and see if it fits.

My finished house have 1040 square feet, because I added up every room's total square feet and I got the answer. Then I check it with the other paper where I wrote down my exact square feet of my whole house and it's the same. Then I times 1040 with 75, because that's how much each square feet costs for a regular room. I used the calculator I got the answer of $78,000. That's how much money I spend I still have $122,000 left, but I also have third answer on the other sheet of paper. The whole perimeter of my house is 142 ft. I did it by counting the walls of my house and adding it together.

Dream House Project



Work Sample & Commentary: *Constructing a Polyhedron*

The task

The teacher gave students the following instructions, and clarified them through an extended discussion:

Look at the polyhedra on this poster. Choose one of them to construct. You will be able to use a diagram of the net in order to help you see how the shapes fit together. You will also:

- Choose a length for the side of the polygons that will be the faces of your polyhedron.
- Use a compass to make a template of the polygons that make the faces of your polyhedron. The main face you will use will be an equilateral triangle.
- Extend the sides to make flaps, and then join the faces together by using rubber bands on the flaps.
- Write about your construction.

The main choices of polyhedra to construct included the regular octahedron (a figure with eight equilateral triangles as faces), the regular icosahedron (a figure with 20 equilateral triangles as faces), and the cuboctahedron (a figure with six squares and eight triangles as faces).

This work sample illustrates a standard-setting performance for the following parts of the standards:

- **M2 d Geometry and Measurement Concepts: Use many types of figures.**
- **M2 f Geometry and Measurement Concepts: Extend and create geometric patterns using concrete and pictorial models.**
- **M2 g Geometry and Measurement Concepts: Use basic ways of estimating and measuring the size of figures and objects in the real world.**
- **M2 i Geometry and Measurement Concepts: Select and use units for estimating and measuring length.**
- **M6 b Mathematical Skills and Tools: Estimate numerically and spatially.**
- **M6 c Mathematical Skills and Tools: Measure accurately.**
- **M6 e Mathematical Skills and Tools: Refer to geometric shapes and terms correctly.**
- **M6 h Mathematical Skills and Tools: Use measuring devices.**

The student in this sample constructed a snub icosadodecahedron. This polyhedron was not offered as a choice for students to construct, because it was considered too complex for elementary students. The snub icosadodecahedron is a polyhedron made of 80 triangles and 12 pentagons. (Its name is derived from the dodecahedron, which is composed of pentagons, and the icosahedron, which is composed of equilateral triangles.)

In order to complete this task, the student voluntarily:

- figured out the net for the polyhedron, with assistance from the teacher;
- received additional instruction from the teacher about how to use a protractor to measure the angles and how to construct a pentagon;
- spent several hours beyond the allotted time for the project, including before and after school and during lunch recesses.

Circumstances of performance

This sample of student work was produced under the following conditions:

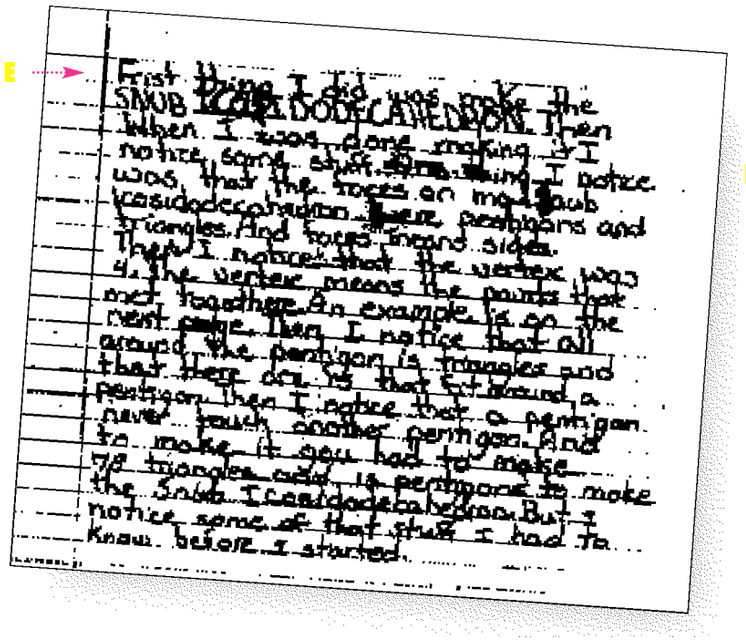
alone	in a group
in class	as homework
with teacher feedback	with peer feedback
timed	opportunity for revision

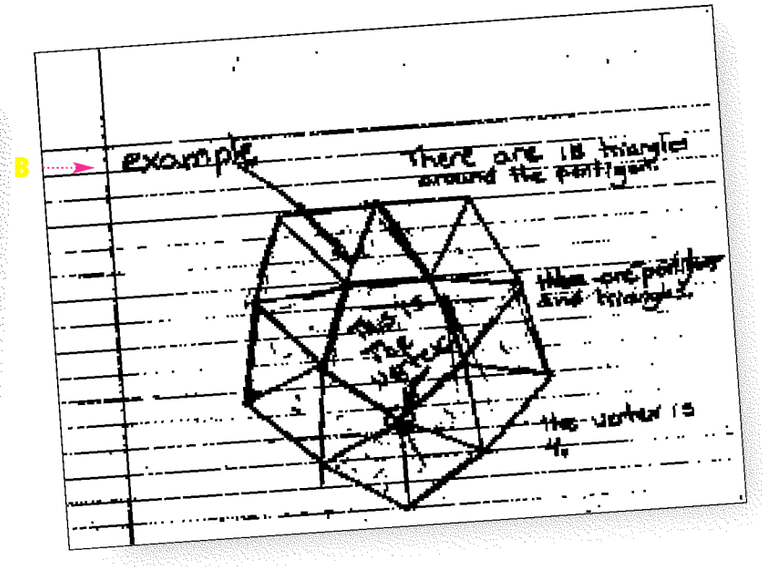
Before beginning this project, students had been working with two dimensional geometry concepts, especially symmetry, congruence, quadrilaterals, triangles, circles, and diameter. They had constructed two dimensional "stained glass windows," according to specifications about the geometry required.

This student's work illustrates parts of the standard for Geometry and Measurement Concepts. In addition, the sample helps to illustrate what it means for an elementary student to exceed the standard in some areas, e.g., the student constructs and works with pentagons, measuring the angles of the pentagon in degrees.

The quotations from the Mathematics performance descriptions in this commentary are excerpted. The complete performance descriptions are shown on pages 60-65.

Constructing a Polyhedron





Arithmetic and Number Concepts
Geometry & Measurement Concepts M2
Function & Algebra Concepts
Statistics & Probability Concepts
Problem Solving & Reasoning
Mathematical Skills & Tools M6
Mathematical Communication
Putting Mathematics to Work

What the work shows

M2 d Geometry and Measurement Concepts: The student uses many types of figures (angles, triangles,…polygons) and identifies the figures by their properties….

A The student used triangles; her use of the pentagon and snub icosadodecahedron represents work that exceeds the standard for this level.

M2 f Geometry and Measurement Concepts: The student extends and creates geometric patterns using concrete and pictorial models.

B The student found a pattern in this polyhedron.

M2 g Geometry and Measurement Concepts: The student uses basic ways of estimating and measuring the size of figures and objects in the real world….

A C In order to construct the triangles and pentagons, the student measured the length of the sides.

M2 i Geometry and Measurement Concepts: The student selects and uses units, both formal and informal as appropriate, for estimating and measuring quantities such as…length….

A The use of inches is appropriate for this level; the use of degrees exceeds the standard for this level.

M6 b Mathematical Skills and Tools: The student estimates numerically and spatially.

A D The successful completion of the icosadodecahedron provides evidence that the student estimated spatially and numerically, as both kinds of estimation were clearly required in the construction.

M6 c Mathematical Skills and Tools: The student measures length…accurately in…the customary… system.

A C

M6 e Mathematical Skills and Tools: The student refers to geometric shapes and terms correctly with concrete objects or drawings, including triangle,…side,…face,…point, line…; and refers with assistance to…polygon, polyhedron, angle, vertex….

E The student refers correctly to angle, triangle, pentagon, and icosadodecahedron. The student appears to have a working understanding of the term "vertex" but has not yet developed a clear definition. (The writing of "4½" is also incorrect in some places.)

M6 h Mathematical Skills and Tools: The student uses…measuring devices…to achieve solutions.

A C The use of a ruler is appropriate for this level; the use of the compass and protractor exceeds the standard.

If this project were written up in a more systematic way, it would illustrate an example of a pure mathematics investigation, one of the projects described in M6.

Several words are misspelled in the written portion of the work. This was a class assignment and was not intended to be further edited for spelling or grammar. The focus of the assignment was the construction. The spelling and grammar errors are reasonable for first draft work, e.g., "notice" for "noticed" and "pentigon" for "pentagon."

Readers who build this figure will discover that there are actually 80 triangles, not 78, as the student reports.

Constructing a Polyhedron