Building directly on the National Council of Teachers of Mathematics (NCTM) Curriculum Standards, the Mathematics performance standards present a balance of conceptual understanding, skills, and problem solving.

The first four standards are the important conceptual areas of mathematics:

**M1** Arithmetic and Number Concepts;
**M2** Geometry and Measurement Concepts;
**M3** Function and Algebra Concepts;
**M4** Statistics and Probability Concepts.

These conceptual understanding standards delineate the important mathematical content for students to learn. To demonstrate understanding in these areas, students need to provide evidence that they have used the concepts in a variety of ways that go beyond recall. Specifically, students show progressively deeper understanding as they use a concept in a range of concrete situations and simple problems, then in conjunction with other concepts in complex problems; as they represent the concept in multiple ways (through numbers, graphs, symbols, diagrams, or words, as appropriate) and explain the concept to another person.

This is not a hard and fast progression, but the concepts included in the first four standards have been carefully selected as those for which the student should demonstrate a robust understanding. These standards make explicit that students should be able to demonstrate understanding of a mathematical concept by using it to solve problems, representing it in multiple ways (through numbers, graphs, symbols, diagrams, or words, as appropriate), and explaining it to someone else. All three ways of demonstrating understanding—use, represent, and explain—are required to meet the conceptual understanding standards.

Establishing separate standards for these areas is a mechanism for highlighting the importance of these areas, but does not imply that they are independent of conceptual understanding. As the work samples that follow illustrate, good work usually provides evidence of both.

Like conceptual understanding, the definition of problem solving is demanding and explicit. Students use mathematical concepts and skills to solve non-routine problems that do not lay out specific and detailed steps to follow; and solve problems that make demands on all three aspects of the solution process—formulation, implementation, and conclusion. These are defined in **M5**, Problem Solving and Reasoning.

The importance of skills has not diminished with the availability of calculators and computers. Rather, the need for mental computation, estimation, and interpretation has increased. The skills in **M6**, Mathematical Skills and Tools, need to be considered in light of the means of assessment. Some skills are so basic and important that students should be able to demonstrate fluency, accurately and automatically; it is reasonable to assess them in an on-demand setting, such as the New Standards reference examination. There are other skills for which students need only demonstrate familiarity rather than fluency. In using and applying such skills they might refer to notes, books, or other students, or they might need to take time out to reconstruct a method they have seen before. It is reasonable to find evidence of these skills in situations where students have ample time, such as in a New Standards portfolio. As the margin note by the examples that follow the performance descriptions indicates, many of the examples are performances that would be expected when students have ample time and access to tools, feedback from peers and the teacher, and an opportunity for revision. This is true for all of the standards, but especially important to recognize with respect to **M6**.
M7 includes two aspects of mathematical communication—using the language of mathematics and communicating about mathematics. Both are important. Communicating about mathematics is about ideas and logical explanation. The travelogue approach adopted by many students in the course of describing their problem solving is not what is intended.

M8 is the requirement that students put many concepts and skills to work in a large-scale project or investigation, at least once each year, beginning in the fourth grade. The types of projects are specified; for each, the student identifies, with the teacher, a clear purpose for the project, what will be accomplished, and how the project involves putting mathematics to work; develops a question and a plan; writes a detailed description of how the project was carried out, including mathematical analysis of the results; and produces a report that includes acknowledgment of assistance received from parents, peers, and teachers.

The examples
The purpose of the examples listed under the performance descriptions is to show what students might do or might have done in achieving the standards, but these examples are not intended as the only ways to demonstrate achievement of the standard. They are meant to illustrate good tasks and they begin to answer the question, “How good is good enough?” “Good enough” means being able to solve problems like these.

Each standard contains several parts. The examples below are cross-referenced to show a rough correspondence between the parts of the standard and the examples. These are not precise matches, and students may successfully accomplish the task using concepts and skills different from those the task designer intended, but the cross-references highlight examples for which a single activity or project may allow students to demonstrate accomplishment of several parts of one or more standards.

The purpose of the examples of student work is to help explain what the standards mean and to elaborate the meaning of a “standard-setting performance.” Few pieces of work are so all-encompassing as to qualify for the statement, “meets the standard.” Rather, each piece of work shows evidence of meeting the requirements of a selected part or parts of a standard. Further, most of these pieces of work provide evidence related to parts of more than one standard. It is essential to look at the commentary to understand just how the work sample helps to illuminate features of the standards.

Resources
We recognize that some of the standards presuppose resources that are not currently available to all students. The New Standards partners have adopted a Social Compact, which says, in part, “Specifically, we pledge to do everything in our power to ensure all students a fair shot at reaching the new performance standards...This means that they will be taught a curriculum that will prepare them for the assessments, that their teachers will have the preparation to enable them to teach it well, and there will be an equitable distribution of the resources the students and their teachers need to succeed.”

The NCTM standards make explicit the need for calculators of increasing sophistication from elementary to high school and ready access to computers. Although a recent National Center for Education Statistics survey confirmed that most schools do not have the facilities to make full use of computers and video, the New Standards partners have made a commitment to create the learning environments where students can develop the knowledge and skills that are delineated here. Thus, M6, Mathematical Skills and Tools, assumes that students have access to computational tools at the level spelled out by NCTM. This is not because we think that all schools are currently equipped to provide the experiences that would enable students to meet these performance standards, but rather that we think that all schools should be equipped to provide these experiences. Indeed, we hope that making these requirements explicit will help those who allocate resources to understand the consequences of their actions in terms of student performance.

The elementary school performance standards are set at a level of performance that is approximately equivalent to the end of fourth grade. It is expected, however, that some students might achieve this level earlier and others later than this grade. The work samples that follow, however, were all done by students who were, in fact, in the fourth grade.
Arithmetic and Number Concepts

The student demonstrates understanding of a mathematical concept by using it to solve problems, representing it in multiple ways (through numbers, graphs, symbols, diagrams, or words, as appropriate), and explaining it to someone else. All three ways of demonstrating understanding—use, represent, and explain—are required to meet this standard.

The student produces evidence that demonstrates understanding of arithmetic and number concepts; that is, the student:

- M1a Adds, subtracts, multiplies, and divides whole numbers, with and without calculators; that is:
  - adds, i.e., joins things together, increases;
  - subtracts, i.e., takes away, compares, finds the difference;
  - multiplies, i.e., uses repeated addition, counts by multiples, combines things that come in groups, makes arrays, uses area models, computes simple scales, uses simple rates;
  - divides, i.e., puts things into groups, shares equally; calculates simple rates;
  - analyzes problem situations and contexts in order to figure out when to add, subtract, multiply, or divide;
  - solves arithmetic problems by relating addition, subtraction, multiplication, and division to one another;
  - computes answers mentally, e.g., 27 + 45, 30 x 4;
  - uses simple concepts of negative numbers, e.g., on a number line, in counting, in temperature, “owing.”

- M1b Demonstrates understanding of the base ten place value system and uses this knowledge to solve arithmetic tasks; that is:
  - counts 1, 10, 100, or 1,000 more than or less than, e.g., 1 less than 10,000, 10 more than 380, 1,000 more than 23,000, 100 less than 9,000;
  - uses knowledge about ones, tens, hundreds, and thousands to figure out answers to multiplication and division tasks, e.g., 36 x 10, 18 x 100, 7 x 1,000, 4,000 ÷ 4.

- M1c Estimates, approximates, rounds off, uses landmark numbers, or uses exact numbers, as appropriate, in calculations.

- M1d Describes and compares quantities by using concrete and real world models of simple fractions; that is:
  - finds simple parts of wholes;
  - recognizes simple fractions as instructions to divide, e.g., ½ of something is the same as dividing something by 4;
  - recognizes the place of fractions on number lines, e.g., in measurement;
  - uses drawings, diagrams, or models to show what the numerator and denominator mean, including when adding like fractions, e.g., ¼ + ⅛, or when showing that ⅓ is more than ¼;
  - uses beginning proportional reasoning and simple ratios, e.g., “about half of the people.”

- M1e Describes and compares quantities by using simple decimals; that is:
  - adds, subtracts, multiplies, and divides money amounts;
  - recognizes relationships among simple fractions, decimals, and percents, i.e., that ¼ is the same as 0.5, and ½ is the same as 50%, with concrete materials, diagrams, and in real world situations, e.g., when discovering the chance of a coin landing on heads or tails.

- M1f Describes and compares quantities by using whole numbers up to 10,000; that is:
  - connects ideas of quantities to the real world, e.g., how many people fit in the school’s cafeteria; how far away is a kilometer;
  - finds, identifies, and sorts numbers by their properties, e.g., odd, even, multiple, square.

Examples of activities through which students might demonstrate conceptual understanding of arithmetic and number include:

- Use examples and drawings to show a third grader who is having trouble understanding multiplication why 3 x 6 = 6 x 3.
- Use base ten blocks and numerals or other models and representations to solve 43 x 38, and show how the numbers get taken apart in the process of solving such a problem, e.g., 43 can become 40 + 3, or 38 can become 40 - 2.
- Figure out how many Valentine’s Day cards would be exchanged in total, if there are 30 students in the class and if everyone gave everyone else one card.
- Draw and explain many different ways to make 263, using tens, hundreds and ones.
- Organize a budget for a project.
- Solve the following problem: “You have a book to read that is 100 pages long. You must read it in five days. How many pages should you read each day?”
- Determine the possible number of objects in a group given this information: when put in groups of three, none is left over; when put in groups of two, one is left over; when put in groups of five, one is left over; there are fewer than 40 objects altogether.
- Make reasonable estimates and then calculate accurately the number of beans in a cup, seeds in a pumpkin, ceiling tiles in a room, and wheels on 37 tricycles.
- Show and explain to a classmate the different coin combinations that make 75¢, excluding pennies.
- Find one possible answer for addends that equal the sum of 8,829, when the digits 1, 2, 3, 4, 5, 6, 7, and 8 are used only once in the solution.
- Analyze seasonal variations in temperature, including negative values.
- Put together fraction pieces to make a whole in different ways, e.g., $\frac{1}{4} + \frac{3}{4} = \frac{1}{2}$.
- Draw diagrams to explain how three pizzas can be shared equally by four people.
- Make approximate counts of different kinds of animals in different zones of a small tide pool area.
- Draw and label diagrams to show the fractional value of each piece of a set of tangram pieces, if all seven pieces (i.e., the whole set) are equal to one whole.
- Pretend you see a highway sign that reads, “Exit A, 1 mile. Exit B, 2½ miles,” and shortly thereafter you see a sign that reads, “Exit A, ½ mile.” Figure out how far away exit B must now be.
- Make up stories that go with number sentences, e.g., $5 \times 9 = 45, 27 \times 6 = ?$.
**Geometry and Measurement Concepts**

The student demonstrates understanding of a mathematical concept by using it to solve problems, representing it in multiple ways (through numbers, graphs, symbols, diagrams, or words, as appropriate), and explaining it to someone else. All three ways of demonstrating understanding—use, represent, and explain—are required to meet this standard.

The student produces evidence that demonstrates understanding of geometry and measurement concepts; that is, the student:

- **M2 a** Gives and responds to directions about location, e.g., by using words such as “in front of,” “right,” and “above.”
- **M2 b** Visualizes and represents two dimensional views of simple rectangular three dimensional shapes, e.g., by showing the front view and side view of a building made of cubes.
- **M2 c** Uses simple two dimensional coordinate systems to find locations on a map, and represent points and simple figures.
- **M2 d** Uses many types of figures (angles, triangles, squares, rectangles, rhombi, parallelograms, quadrilaterals, polygons, prisms, pyramids, cubes, circles, and spheres) and identifies the figures by their properties, e.g., symmetry, number of faces, two- or three-dimensionality, no right angles.
- **M2 e** Solves problems by showing relationships between and among figures, e.g., using congruence and similarity, and using transformations including flips, slides, and rotations.
- **M2 f** Extends and creates geometric patterns using concrete and pictorial models.
- **M2 g** Uses basic ways of estimating and measuring the size of figures and objects in the real world, including length, width, perimeter, and area.
- **M2 h** Uses models to reason about the relationship between the perimeter and area of rectangles in simple situations.
- **M2 i** Selects and uses units, both formal and informal as appropriate, for estimating and measuring quantities such as weight, length, area, volume, and time.
- **M2 j** Carries out simple unit conversions, such as between cm and m, and between hours and minutes.
- **M2 k** Uses scales in maps, and uses, measures, and creates scales for rectangular scale drawings based on work with concrete models and graph paper.

**Examples of activities through which students might demonstrate conceptual understanding of geometry and measurement include:**

- Locate a point on a map using its distance away from another given point, the map’s scale, and compass point directions, e.g., N, NE, E, SE, etc. **2a, 2c, 2k**
- Given views of three faces of a three dimensional figure made of stacked cubes, represent the views of the other faces and figure out the total number of cubes used to construct the figure. **2b**
- Explain and show why a square is not a cube. **2d**
- Describe a shape from among the following, using as few attributes as possible to distinguish it from the others: right triangle, equilateral triangle, trapezoid, and parallelogram. **2d**
- Design and label a quilt square which includes two lines of symmetry, four congruent shapes, and two similar shapes; then make a final version at twice the original size, i.e., a “two to one” scale, and add color while maintaining symmetry. **2d, 2e, 2f, 2k**

Draw at each of three consecutive 90° rotations in a clockwise direction. **2e**

Find all the shapes that can be made with five squares if the sides touch completely. **2e**

Figure out the approximate area and perimeter of the bottom of a shoe. **2g**

Solve the following problem, using concrete materials as appropriate: “Given a diagram of a fenced-in, rectangular garden plot with dimensions three meters by eight meters, find its area and perimeter; design a second garden plot using less fencing, but providing greater area; design a third plot using more fencing, but providing less area. Of all possible rectangular designs using the original amount of fencing, which provides the greatest area?” **2g, 2h, 2i**

Put five objects, such as books, rocks, or pumpkins, in rank order by weight, first by estimating and then by measuring exactly. **2i**

Figure out exactly how much time would you gain for the completion of a project by waking up at 6:15 rather than 7:00 for five school days. **2i, 2j**
M Function and Algebra Concepts

The student demonstrates understanding of a mathematical concept by using it to solve problems, representing it in multiple ways (through numbers, graphs, symbols, diagrams, or words, as appropriate), and explaining it to someone else. All three ways of demonstrating understanding—use, represent, and explain—are required to meet this standard.

The student produces evidence that demonstrates understanding of function and algebra concepts; that is, the student:
M3a Uses linear patterns to solve problems; that is:
• shows how one quantity determines another in a linear (“repeating”) pattern, i.e., describes, extends, and recognizes the linear pattern by its rule, such as, the total number of legs on a given number of horses can be calculated by counting by fours;
• shows how one quantity determines another quantity in a functional relationship based on a linear pattern, e.g., for the “number of people and total number of eyes,” figure out how many eyes 100 people have all together.
M3b Builds iterations of simple non-linear patterns, including multiplicative and squaring patterns (e.g., “growing” patterns) with concrete materials, and recognizes that these patterns are not linear.
M3c Uses the understanding that an equality relationship between two quantities remains the same as long as the same change is made to both quantities.
M3d Uses letters, boxes, or other symbols to stand for any number, measured quantity, or object in simple situations with concrete materials, i.e., demonstrates understanding and use of a beginning concept of a variable.

Examples of activities through which students might demonstrate conceptual understanding of functions and algebra include:
Find, make, and describe linear patterns on the 99-chart, e.g., 4, 14, 24, 34, 43.
Given the situation described in the Christmas carol, “The Twelve Days of Christmas,” determine how many gifts would have been given in total; describe any pattern you notice that helps in solving the problem.
Show how the letters “aah, aah, aah...” can represent the pattern “metal, metal, plastic, ..., ‘leaf, leaf, rock, ...” or many other patterns.
Solve the following problem: When building a staircase out of cubes, one step uses a total of one cube, two steps a total of three cubes, three steps a total of six cubes, etc. Find how many cubes are used for six steps, nine steps, n steps. Describe the pattern.
Observe and record, in a two-column table, multiplicative patterns with concrete materials, e.g., how many regions are produced by increasing the numbers of folds of paper; and recognize that this type of pattern does not proceed in a linear way, i.e., not 2, 4, 6, 8, etc., but 2, 4, 8, 16, etc.
Build the fourth, fifth, and sixth iterations in the following sequence:

M4 Statistics and Probability Concepts

The student demonstrates understanding of a mathematical concept by using it to solve problems, representing it in multiple ways (through numbers, graphs, symbols, diagrams, or words), and explaining it to someone else. All three ways of demonstrating understanding—use, represent, and explain—are required to meet this standard.

The student produces evidence that demonstrates understanding of statistics and probability concepts in the following areas; that is, the student:
M4a Collects and organizes data to answer a question or test a hypothesis by comparing sets of data.
M4b Displays data in line plots, graphs, tables, and charts.
M4c Makes statements and draws simple conclusions based on data; that is:
• reads data in line plots, graphs, tables, and charts;
• compares data in order to make true statements, e.g., “seven plants grew at least 5 cm”;
• identifies and uses the mode necessary for making true statements, e.g., “more people chose red”;
• makes true statements based on a simple concept of average (median and mean), for a small sample size and where the situation is made evident with concrete materials or clear representations;
• interprets data to determine the reasonableness of statements about the data, e.g., “twice as often,” “three times faster”;
• uses data, including statements about the data, to make a simple concluding statement about a situation, e.g., “This kind of plant grows better near sunlight because the seven plants that were near the window grew at least 5 cm.”
M4d Gathers data about an entire group or by sampling group members to understand the concept of sample, i.e., that a large sample leads to more reliable information, e.g., when flipping coins.
M4e Predicts results, analyzes data, and finds out why some results are more likely, less likely, or equally likely.
M4f Finds all possible combinations and arrangements within certain constraints involving a limited number of variables.

Examples of activities through which students might demonstrate conceptual understanding of statistics and probability include:
Generate survey questions, such as: How many people are in your family? What is your hair color? What do you do at recess? Predict the survey results, carry out the survey, graph the data, and write true statements about the data.
Make conjectures, after sampling the particular situation, about how many raisins are in a given box taken from a set of boxes or what colors of tiles are in a bag.
Investigate the temperature over the entire school year.
Investigate the possible and likely or unlikely outcomes when rolling two number cubes and recording the sums.
Design spinners with regions red, blue, and green, according to the following instructions: Spinner #1: red will certainly win; Spinner #2: red cannot win; Spinner #3: red is likely to win; Spinner #4: red, blue and green are equally likely to win; Spinner #5: red or blue will probably win.
Find all of the different combinations possible using three ice cream flavors, two sauces, and one topping; then discover which would allow the most new combinations—an additional ice cream flavor or a new topping.

(Balanced Assessment)
Problem Solving and Reasoning

The student demonstrates logical reasoning throughout work in mathematics, i.e., concepts and skills, problem solving, and projects; demonstrates problem solving by using mathematical concepts and skills to solve non-routine problems that do not lay out specific and detailed steps to follow; and solves problems that make demands on all three aspects of the solution process—formulation, implementation, and conclusion.

Formulation

**M5 a** Given the basic statement of a problem situation, the student:
- makes the important decisions about the approach, materials, and strategies to use, i.e., does not merely fill in a given chart, use a pre-specified manipulative, or go through a predetermined set of steps;
- uses previously learned strategies, skills, knowledge, and concepts to make decisions;
- uses strategies, such as using manipulatives or drawing sketches, to model problems.

Implementation

**M5 b** The student makes the basic choices involved in planning and carrying out a solution; that is, the student:
- makes up and uses a variety of strategies and approaches to solving problems and uses or learns approaches that other people use, as appropriate;
- makes connections among concepts in order to solve problems;
- solves problems in ways that make sense and explains why these ways make sense, e.g., defends the reasoning, explains the solution.

Conclusion

**M5 c** The student moves beyond a particular problem by making connections, extensions, and/or generalizations; for example, the student:
- explains a pattern that can be used in similar situations;
- explains how the problem is similar to other problems he or she has solved;
- explains how the mathematics used in the problem is like other concepts in mathematics;
- explains how the problem solution can be applied to other school subjects and in real world situations;
- makes the solution into a general rule that applies to other circumstances.

Examples of activities through which students might demonstrate facility with problem solving and reasoning include:

Suppose you are given a string that is sixteen inches long. If you cut or fold it in any two places, will it always make a triangle? **5a, 5b, 5c**

Figure out how many handshakes there will be altogether if five people in a room shake each other’s hand just once. **5a, 5b, 5c**

Prove or disprove a classmate’s claim that ¼ and ½ are really the same because both have one piece missing. **5a, 5b, 5c**

Given an unopened bag of chocolate chip cookies, with cookies arranged in rows, show and explain whether the manufacturer’s claim of “More than 1,000 chips in every bag!” is reasonable. **5a, 5b, 5c**

Figure out the value of each tangram piece if the small tangram triangle is worth one cent and the value is proportional to the area. **5a, 5b, 5c**

For the sum of 8,829, find one possible answer for addends when the digits 1, 2, 3, 4, 5, 6, 7, and 8 are used only once in the solution. **5a, 5b, 5c**
Mathematical Skills and Tools

The student demonstrates fluency with basic and important skills by using these skills accurately and automatically, and demonstrates practical competence and persistence with other skills by using them effectively to accomplish a task, perhaps referring to notes, books, or other students, perhaps working to reconstruct a method; that is, the student:

M6a Adds, subtracts, multiplies, and divides whole numbers correctly; that is:
- knows single digit addition, subtraction, multiplication, and division facts;
- adds and subtracts numbers with several digits;
- multiples and divides numbers with one or two digits;
- multiples and divides three digit numbers by one digit numbers.

M6b Estimates numerically and spatially.

M6c Measures length, area, perimeter, circumference, diameter, height, weight, and volume accurately in both the customary and metric systems.

M6d Computes time (in hours and minutes) and money (in dollars and cents).

M6e Refers to geometric shapes and terms correctly with concrete objects or drawings, including triangle, square, rectangle, side, edge, face, cube, point, line, perimeter, area, and circle; and refers with assistance to rhombus, parallelogram, quadrilateral, polygon, polyhedron, angle, vertex, volume, diameter, circumference, sphere, prism, and pyramid.

M6f Uses +, x, ÷, /, √, S, %, and . (decimal point) correctly in number sentences and expressions.

M6g Reads, creates, and represents data on line plots, charts, tables, diagrams, bar graph, simple circle graphs, and coordinate graphs.

M6h Uses recall, mental computations, pencil and paper, measuring devices, mathematics texts, manipulatives, calculators, computers, and advice from peers, as appropriate, to achieve solutions; that is, uses measuring devices, graded appropriately for given situations, such as rulers (customary to the 1/16 inch; metric to the millimeter); graph paper (customary to the inch or half-inch; metric to the centimeter); measuring cups (customary to the ounce; metric to the milliliter), and scales (customary to the pound or ounce; metric to the kilogram or gram).

Examples of activities through which students might demonstrate facility with mathematical skills and tools include:

- Know that 6 x 7 = 42. 6a
- Use an efficient mental approach to estimate the total cost of items costing $2.94, $1.28 and $0.74. 6a, 6b, 6d, 6h
- Mentally add two digit numbers correctly during problem solving. 6a, 6b
- Decide whether to use a calculator, paper and pencil or mental arithmetic to figure out 6 x 6,000. 6a, 6h
- Figure out rectangular areas correctly when designing a floor plan for a "dream house." 6b, 6c
- Measure the circumference of a pumpkin accurately by using a piece of yarn, and then laying it next to a ruler. 6c, 6e
- Use a calculator to check the arithmetic in a project.
- Use a table to record functions such as how many chairs fit at how many tables. 6g
- Use a Venn diagram to record students who wore a sweater to school and students who walked to school. 6g
- Make a bar graph or simple circle graph to show how many students like different kinds of vegetables. 6g, 6f

Mathematical Communication

The student uses the language of mathematics, its symbols, notation, graphs, and expressions, to communicate through reading, writing, speaking, and listening, and communicates about mathematics by describing mathematical ideas and concepts and explaining reasoning and results; that is, the student:

M7a Uses appropriate mathematical terms, vocabulary, and language, based on prior conceptual work.

M7b Shows mathematical ideas in a variety of ways, including words, numbers, symbols, pictures, charts, graphs, tables, diagrams, and models.

M7c Explains solutions to problems clearly and logically, and supports solutions with evidence, in both oral and written work.

M7d Considers purpose and audience when communicating about mathematics.

M7e Comprehends mathematics from reading assignments and from other sources.

Examples of activities through which students might demonstrate facility with mathematical communication include:

- Use words, numbers, or diagrams to explain how to take numbers apart in order to solve problems using mental math, e.g., \(25 \times 6\). One way is \(20 \times 6 = 120\), and \(5 \times 6 = 30\); \(120 + 30 = 150\). Or, \(25 \times 4 = 100\), and \(25 \times 2 = 50\), and so the answer is 150 because \(100 + 50 = 150\). 7a, 7b, 7c
- Explain why \(34 + 17 = 34 + 17\) to a first grader or to a visitor from outer space. 7a, 7b, 7c, 7d
- Show \(\frac{1}{2} \div \frac{1}{3}\) in pictures and diagrams so a younger student could understand the sum. 7a, 7b, 7c, 7d
- Use clear, correct mathematical language to describe a shape composed of several geometric solids so that it could be reproduced exactly by a person who cannot see the shape. 7a, 7b, 7c, 7d
- Represent survey data about student school lunch preferences in graphical, written, and numerical form in order to make a clear and effective recommendation to kitchen staff about menu changes. 7a, 7b, 7c, 7d
- Give an oral presentation of a preliminary investigation of classification of shapes, in order to get peer feedback; then revise the classification scheme to make it clearer. 7a, 7b, 7c, 7d, 7e
- Prepare a report, including graphs, charts, and diagrams, on the optimal number and location of recycling containers, based on data from the classroom and the entire school. 7a, 7b, 7c, 7d, 7e, 7f, 7g, 7h, 7i, 7j, 7k, 7l, 7m, 7n, 7o, 7p, 7q, 7r, 7s, 7t, 7u, 7v, 7w, 7x, 7y, 7z
Putting Mathematics to Work

The student conducts at least one large scale project each year, beginning in fourth grade, drawn from the following kinds and, over the course of elementary school, conducts projects drawn from at least two of the kinds.

A single project may draw on more than one kind.

**M8 a** Data study, in which the student:
- develops a question and a hypothesis in a situation where data could help make a decision or recommendation;
- decides on a group or groups to be sampled and makes predictions of the results, with specific percents, fractions, or numbers;
- collects, represents, and displays data in order to help make the decision or recommendation; compares the results with the predictions;
- writes a report that includes recommendations supported by diagrams, charts, and graphs, and acknowledges assistance received from parents, peers, and teachers.

**M8 b** Science study, in which the student:
- decides on a specific science question to study and identifies the mathematics that will be used, e.g., measurement;
- develops a prediction (a hypothesis) and develops procedures to test the hypothesis;
- collects and records data, represents and displays data, and compares results with predictions;
- writes a report that compares the results with the hypothesis; supports the results with diagrams, charts, and graphs; acknowledges assistance received from parents, peers, and teachers.

**M8 c** Design of a physical structure, in which the student:
- decides on a structure to design, the size and budget constraints, and the scale of design;
- makes a first draft of the design, and revises and improves the design in response to input from peers and teachers;
- makes a final draft and report of the design, drawn and written so that another person could make the structure; acknowledges assistance received from parents, peers, and teachers.

**M8 d** Management and planning, in which the student:
- decides on what to manage or plan, and the criteria to be used to see if the plan worked;
- identifies unexpected events that could disrupt the plan and further plans for such contingencies;
- identifies resources needed, e.g., materials, money, time, space, and other people;
- writes a detailed plan and revises and improves the plan in response to feedback from peers and teachers;
- carries out the plan (optional);
- writes a report on the plan that includes resources, budget, and schedule, and acknowledges assistance received from parents, peers, and teachers.

**M8 e** Pure mathematics investigation, in which the student:
- decides on the area of mathematics to investigate, e.g., numbers, shapes, patterns;
- describes a question or concept to investigate;
- decides on representations that will be used, e.g., numbers, symbols, diagrams, shapes, or physical models;
- carries out the investigation;
- writes a report that includes any generalizations draw from the investigation, and acknowledges assistance received from parents, peers, and teachers.

**Examples of projects include:**
Develop questions and a hypothesis for a study of students’ diets; collect, organize, display, and analyze the data; and make recommendations to the school community based on the data.

8a, 8b, 8c, 8d, 8e

Compare the growth of a set of plants under a variety of conditions, e.g., amount of water, fertilizer, duration and exposure to sunlight. 8b, 8c

Make a design for a tree house that accounts for physical and financial constraints. 8e, 8c

Plan a class camping trip, including making a schedule, researching costs and facilities, developing a budget. 8d, 8e, 8f, 8g, 8h, 8i

Plan and conduct a probability study that compares the results from three different spinners, e.g.,

1 2 1 2 1

8e
Work Sample & Commentary: Sharing 25

The task

The following written prompt appeared on an examination:

In each situation below, four friends want to “share 25” as equally as possible. Show or explain how to “share 25” in each situation.

1. Four friends shared 25 balloons as equally as possible.
2. Four friends shared $25 as equally as possible.
3. Four friends shared 25 cookies as equally as possible.

Circumstances of performance

This sample of student work was produced under the following conditions:

- alone
- in a group
- in class
- as homework
- with teacher feedback
- with peer feedback
- timed
- opportunity for revision

This five minute task was part of a field test for the New Standards Reference Examination: Mathematics (Elementary).

What the work shows

M1 a Arithmetic and Number Concepts: The student adds, subtracts, multiplies, and divides whole numbers, with and without calculators; that is:

- divides, i.e., puts things into groups, shares equally....

A B C

M1 c Arithmetic and Number Concepts: Estimate, approximate, round off, use landmark numbers, or use exact numbers in calculations.

M1 d Arithmetic and Number Concepts: Describe and compare quantities by using simple fractions.

M1 e Arithmetic and Number Concepts: Describe and compare quantities by using simple decimals.

M6 a Mathematical Skills and Tools: Add, subtract, multiply, and divide whole numbers correctly.

M6 d Mathematical Skills and Tools: Compute time and money.

M6 f Mathematical Skills and Tools: Use +, -, *, /, \, $, %, and . (decimal point) correctly in number sentences and expressions.
Arithmetic and Number Concepts: The student estimates, approximates, rounds off, uses landmark numbers, or uses exact numbers, as appropriate, in calculations.

A B C The correct answers demonstrate rounding off or use of exact number, as appropriate, in each situation.

Arithmetic and Number Concepts: The student describes and compares quantities by using concrete and real world models of simple fractions; that is:

- finds simple parts of wholes....
  B C

Arithmetic and Number Concepts: The student describes and compares quantities by using simple decimals; that is:

- adds, subtracts, multiplies, and divides money amounts....
  B

Mathematical Skills and Tools: The student adds, subtracts, multiplies, and divides whole numbers correctly....

Mathematical Skills and Tools: The student computes...money (in dollars and cents).

B

Mathematical Skills and Tools: The student uses...and . (decimal point) correctly....

B
Work Sample & Commentary: Arithmetic

The tasks
These tasks all focus on the same arithmetic and number concepts and skills, although they were drawn from four different classrooms.

In Sample 1, the teacher included the following instructions on a classroom test:

Using what you know about multiplication, find the answer to the equation below. Show all your work and clearly explain how you got your answer.

63 \times 46 =

In Sample 2, the teacher gave the following written instructions as part of a class exercise:

Solve each problem two ways.

522 - 367 =

87 \times 9 =

These work samples illustrate standard-setting performances for the following parts of the standards:

M1 a Arithmetic and Number Concepts: Add, subtract, multiply, and divide whole numbers.

M1 b Arithmetic and Number Concepts: Demonstrate understanding of the base ten value system and use this knowledge to solve arithmetic tasks.

M1 c Arithmetic and Number Concepts: Estimate, approximate, round off, use landmark numbers, or use exact numbers in calculations.

M1 d Arithmetic and Number Concepts: Describe and compare quantities by using simple fractions.

M6 a Mathematical Skills and Tools: Add, subtract, multiply, and divide whole numbers correctly.

M6 b Mathematical Skills and Tools: Use recall, mental computations, and pencil and paper to achieve solutions.

M7 a Mathematical Communication: Use appropriate mathematical terms, vocabulary, and language.

M7 b Mathematical Communication: Show mathematical ideas in a variety of ways.

M7 c Mathematical Communication: Explain solutions to problems clearly and logically.

In Sample 3, the teacher gave the following oral instructions at the end of a long unit of work:

Make a catalogue of all the ways the class has come up with to multiply large numbers.

Make up a two digit by two digit multiplication problem and use these ways to find the answer.

In Sample 4, a specialist read the following written instructions to an individual student:

Four children had three bags of M & M’s.

They decided to open all three bags of candy and share the M & M’s fairly.

There were 52 M & M candies in each bag.

How many M & M candies did each child get?

Circumstances of performance
These samples of student work were produced under the following conditions:

alone in a group
in class as homework
with teacher feedback with peer feedback
timed opportunity for revision

Sample 1 was part of a classroom test. For Samples 2 and 3, students were permitted to talk as they worked. Sample 4 was produced when a specialist interviewed the student as the student was completing the work.
Sample 1 was produced before the class had received any instruction about two digit by two digit multiplication, although they had experience in developing strategies for simpler computation.

Sample 2 was produced in a classroom where students' computation strategies were regularly shared, and several were posted on the wall.

Sample 3 was produced after a long unit of work in which students created, developed, and learned a range of strategies for single and double digit multiplication.

Sample 4 was produced before the class had received any instruction about division, but they had extensive experience with developing and explaining strategies for other computation problems.

What the work shows

**Arithmetic and Number Concepts:** The student adds, subtracts, multiplies, and divides whole numbers, with and without calculators; that is:
- adds, i.e., joins things together, increases.
- subtracts, i.e., takes away, compares, finds the difference.
- multiplies, i.e., uses repeated addition, counts by multiples, combines things that come in groups, makes arrays.

**A** The student combined doubling with repeated addition.

**C** The student used repeated addition of 85.

**H** The student showed how the problem could be solved by making 62 circles with 85 stars in each circle.

**I** The student drew a base ten block array.

**J** The student “split” 80 into 40 and 40 in order to multiply.

**E** The student began to divide 156 by 4 by putting down 20 four times.

**K** The student analyzed problem situations and contexts in order to figure out when to add, subtract, multiply, or divide.

**E** In Sample 4, the instructions give no indication about how arithmetic can be used to solve the problem: the student figured it all out.

**A** The students related addition to multiplication.
Arithmetic

These parts of the work demonstrate using addition to solve a subtraction problem.

The student counted by twos in the process of multiplying.

The student counted by multiples to solve a division problem.

computes answers mentally, e.g., 27 + 45, 30 x 4.

All students used mental computation effectively throughout all of their work.

Arithmetic and Number Concepts: The student demonstrates understanding of the base ten place value system and uses this knowledge to solve arithmetic tasks; that is:

• counts 1, 10, 100, or 1,000 more than or less than, e.g., 1 less than 10,000, 10 more than 380, 1,000 more than 23,000, 100 less than 9,000.

The student counted 3, 30, and 100 more.

The student counted by twenties, tens, and ones.

Each sample includes evidence that demonstrates this kind of understanding.

uses knowledge about ones, tens, hundreds, and thousands to figure out answers to multiplication and division tasks, e.g., 36 x 10, 18 x 100, 7 x 1,000, 4,000 ÷ 4.

The students broke the numbers apart, e.g., 40 x 9; 60 x 80; 3 x 50.

Arithmetic and Number Concepts: The student estimates, approximates, rounds off, uses landmark numbers, or uses exact numbers, as appropriate, in calculations.

The student used landmark numbers to go from 367 to 370 to 400 to 500.

The student rounded off in order to make the problem easier.

Each sample demonstrates the use of exact numbers to finish the tasks.

Arithmetic and Number Concepts: The student describes and compares quantities by using concrete and real world models of simple fractions; that is:

finds simple parts of wholes.

The student applied concrete knowledge to split “80 in half.”
M6 Mathematical Skills and Tools: The student adds, subtracts, multiplies, and divides whole numbers correctly; that is:

- knows single digit addition, subtraction, multiplication, and division facts.
- adds and subtracts numbers with several digits.
- multiplies and divides numbers with one or two digits.
- multiplies and divides numbers three digit numbers by one digit numbers.
- multiplies and divides numbers using mental computations, pencil and paper...to achieve solutions.

M6 Mathematical Communication: The student uses correct use of symbols. This is also evident throughout the samples.

M6 Mathematical Communication: The student uses recall, mental computations, pencil and paper...to achieve solutions.

M7 Mathematical Communication: The student uses appropriate mathematical terms, vocabulary, and language, based on prior conceptual work.

M7 Mathematical Communication: The student shows mathematical ideas in a variety of ways, including words, numbers, symbols, pictures, charts, graphs, tables, diagrams, and models.

M7 Mathematical Communication: The student explains solutions to problems clearly and logically, and supports solutions with evidence, in both oral and written work.

In Sample 3, the student spelled a few words inconsistently, e.g., “multiplied” is correct while “multiplyed” is not; and misspelled others, e.g., “Catalogue” instead of “Catalogue.” All of the samples were taken from work done in class which was not edited for spelling.
Work Sample & Commentary: 3-D to 2-D

The tasks
These work samples are drawn from two different classrooms, but focus on the same concept of visualizing and representing three dimensional objects in two dimensions.

In Sample 1, the teacher gave the following written instruction to the students:
André built a shape with blocks. What would be the front view?

In Sample 2, the teacher gave the students Student Sheets 15 and 16 (from Seeing Solids and Silhouettes, a book in the series, Investigations in Number, Data and Space, Dale Seymour Publications, 1995) which had these written instructions:
Choose two or three buildings. Draw their silhouettes from the front, top, and right side. (Student Sheet 15)
Draw the front, top, and side silhouettes for three cubic buildings. Put the number of the building above its silhouettes. (Student Sheet 16)

Circumstances of performance
These samples of student work were produced under the following conditions:

alone in a group as homework with peer feedback opportunity for revision
in class timed
with teacher feedback

Both students were developing a New Standards Elementary Mathematics Portfolio. Because of this, they knew the criteria in this system for work that shows conceptual understanding. The criteria require the student to use, represent, and explain the concept. The students used the tasks as opportunities to show what they understood about the concept of visualizing and representing three dimensional objects in two dimensions.

Sample 1 was completed in Spanish in a bilingual classroom. The translation was provided by the teacher.

What the work shows

M2a Geometry and Measurement Concepts: The student gives and responds to directions about location, e.g., by using words such as “in front of,” “right,” and “above.”

AB Both students used location words in their explanations of their thinking and understanding.

M2b Geometry and Measurement Concepts: The student visualizes and represents two dimensional views of simple rectangular three dimensional shapes, e.g., by showing the front view and side view of a building made of cubes.

CD Both students produced two dimensional views to represent three dimensional objects.

ESample 2 includes a small mistake that does not detract from the evidence of an overall understanding of this concept.
Sample 1

Andre built a shape with blocks. What would be the front view?

Translation

The reason why I think this is because I imagined turning around the form without seeing the other cubes remaining. I know this is right because from the left to the right there are 3 cubes. And on top of the 3 cubes there are 3 other cubes so from the right to the left there are a total of 6 cubes. And on the right there is one cube above. And on the left there is one cube above diagonal. So in front there are a total of 8 cubes.

M2 d Geometry and Measurement Concepts: The student uses many types of figures (…squares,…cubes…) and identifies the figures by their properties, e.g., symmetry, number of faces, two- or three-dimensionality, no right angles.
3-D to 2-D

Sample 2

March 8, 96

Spatial relationships are really interesting, fun, and exciting. I built these shapes with inter-locking cubes. I looked at things from the front, top, and side. This is called perspective. Perspective means looking at things from different views or angles.

For example

TOP View

More Cube Buildings
Choose two or three buildings. Draw their silhouettes from the front, top, and right side.

1. 

2. 

3. 

4. 

5. 

6. 

More Cube Silhouettes
Draw the front, top, and side silhouettes for three cube buildings. Put the number of the building above its silhouettes.

Building number 1

Building number 2

Building number 3

Building number 4

Building number 5

Building number 6

Mathematical Communication: The student uses appropriate mathematical terms, vocabulary, and language, based on prior conceptual work.

A Sample 1 includes appropriate use of location words (e.g., “from the right to the left”) and shape words (“cube”).

B Sample 2 includes a good elementary definition of perspective, supported with a clearly explained example.

F Sample 2 includes some unnecessary and superfluous communication. While this kind of communication is fairly common for students as they develop their capacity to explain their thinking, reasoning, and understanding, it does not add any mathematical communication to the response.
Work Sample & Commentary: Patterns

The task
The teacher asked students to respond in their mathematics journals to the following question: What is a mathematical pattern?

Circumstances of performance
This sample of student work was produced under the following conditions:
- alone
- in class
- with teacher feedback
- timed
  - in a group
  - as homework
  - with peer feedback
  - opportunity for revision

What the work shows

M3a Function and Algebra Concepts: The student uses linear patterns to solve problems; that is, shows how one quantity determines another in a linear (“repeating”) pattern, i.e., describes, extends, and recognizes the linear pattern by its rule, such as the total number of legs on a given number of horses can be calculated by counting by fours.

A B The student explained what patterns are, and noted that a pattern can be linear, i.e., that it “may or may not repeat itself.” The student gave an example of a linear pattern and extended it.

M3b Function and Algebra Concepts: The student builds iterations of simple non-linear patterns, including multiplicative and squaring patterns (e.g., “growing” patterns) with concrete materials, and recognizes that these patterns are not linear.

A The student explained what patterns are, and noted that a pattern can “grow or lower itself,” i.e., be non-linear.

C D The student went on to give two examples of different kinds of non-linear patterns: one in which the numbers get larger by adding the next odd number in sequence; the other in which the numbers get larger by adding “numbers in sequence.”

M7a Mathematical Communication: The student uses appropriate mathematical terms, vocabulary, and language, based on prior conceptual work.

A B C D These parts of the work provide evidence of appropriate use of mathematical terms and vocabulary. This evidence is supported throughout the work.

This work sample illustrates a standard-setting performance for the following parts of the standards:

M3a Function and Algebra Concepts: Use linear patterns to solve problems.


M7a Mathematical Communication: Use appropriate mathematical terms, vocabulary, and language.
**Work Sample & Commentary: Pumpkin Activity**

**The task**
The teacher gave these instructions:
- Estimate and then measure the height, diameter, circumference, and weight of your group’s pumpkin. Use the pieces of yarn, rulers, and a bathroom scale.
- Record your estimated and actual measurements in a clear and organized way.
- Explain how you made the measurements of each feature you are measuring.
- Estimate the number of seeds in your pumpkin, decide on a counting method, and then count the seeds.
- Record what true statements or conclusions you can make about the data from all eight class pumpkins.

**Circumstances of performance**
This sample of student work was produced under the following conditions:
- alone
- in a group
- in class
- as homework
- with teacher feedback
- with peer feedback
- timed
- opportunity for revision

While students shared pumpkins in groups of four, they made their estimates and recorded the data individually. The teacher verified that students measured correctly.

**What the work shows**

**M2 g** Geometry and Measurement Concepts: The student uses basic ways of estimating and measuring the size of figures and objects in the real world including length, width, perimeter, and area.

**A** The student estimated, measured, recorded, and described the weight, diameter, circumference, and height of a pumpkin.

**M4 b** Statistics and Probability Concepts: The student displays data in...tables and charts.

**B C** The student produced a chart and a table that provide clear displays of data about the group’s estimates and measurements and about the data from the whole class.

---

**This work sample illustrates a standard-setting performance for the following parts of the standards:**

**M2 g** Geometry and Measurement Concepts: Use basic ways of estimating and measuring the size of figures and objects in the real world.

**M4 b** Statistics and Probability Concepts: Display data.

**M4 c** Statistics and Probability Concepts: Make statements and draw simple conclusions based on data.

**M6 b** Mathematical Skills and Tools: Estimate numerically and spatially.

**M6 c** Mathematical Skills and Tools: Measure accurately.

**M6 g** Mathematical Skills and Tools: Read, create, and represent data.

**M6 h** Mathematical Skills and Tools: Use measuring devices.

**M7 a** Mathematical Communication: Use appropriate mathematical terms, vocabulary, and language.

**M7 b** Mathematical Communication: Show mathematical ideas in a variety of ways.
Pumpkin Activity

M4  Statistics and Probability Concepts: The student makes statements...based on data; that is, reads data in...tables and charts; compares data in order to make true statements....

D  The student read the data about the class pumpkins in order to make comparisons and produce true statements about them.

M6  Mathematical Skills and Tools: The student estimates numerically and spatially.

E  The student estimated weight, diameter, circumference, and height.

M6  Mathematical Skills and Tools: The student measures length,...circumference, diameter,...weight accurately....

F  The student measured weight, diameter, circumference, and height accurately. These measurements were verified by the teacher during the activity.

M6  Mathematical Skills and Tools: The student reads, creates, and represents data on...charts, tables....

B  The chart and table are correctly drawn.

M4  Mathematical Skills and Tools: The student uses...measuring devices...to achieve solutions.

A  The student used rulers and yarn to find measurements.

M7  Mathematical Communication: The student uses appropriate mathematical terms, vocabulary, and language, based on prior conceptual work.

D  True statements based on the data are communicated clearly.

M7  Mathematical Communication: The student shows mathematical ideas in a variety of ways, including words, numbers, symbols, pictures, charts, graphs, tables, diagrams, and models.

A  The process of measurement is explained in multiple ways (words, diagrams, symbolic notation) and the data are represented in a clear and easily interpreted chart and table.

B  The work includes some misspelled words. This sample represents work done in class which was not further edited for spelling. The errors occur in words that represented relatively new concepts for the student, e.g., “estiment” for “estimate” and “circumference” for “circumference.”

The following table shows the number of seeds in each pumpkin reported by table groups. The student referred to these data in the “Conclusions” section of the response.

<table>
<thead>
<tr>
<th></th>
<th>Table 1</th>
<th>Table 2</th>
<th>Table 3</th>
<th>Table 4</th>
<th>Table 5</th>
<th>Table 6</th>
<th>Table 7</th>
<th>Table 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of seeds</td>
<td>369</td>
<td>600</td>
<td>824</td>
<td>738</td>
<td>793</td>
<td>562</td>
<td>233</td>
<td>486</td>
</tr>
</tbody>
</table>
Work Sample & Commentary: Height Measurement

Statistics

The task
The teacher gave the following instructions:

• Collect everyone’s height measurement in inches.
• Make a line plot with the data.
• Write about what you noticed about the data.

Circumstances of performance
This sample of student work was produced under the following conditions:

alone in a group
in class as homework
with teacher feedback with peer feedback
timed opportunity for revision

What the work shows

M4 a Statistics and Probability Concepts: The student collects and organizes data to answer a question....

A B The student began with the question of how tall her classmates were, and collected and organized data to answer this question.

M4 b Statistics and Probability Concepts: The student displays data in line plots....

B

M4 c Statistics and Probability Concepts: The student makes statements and draws simple conclusions based on data; that is, the student:

• reads data in line plots....

B

• compares data in order to make true statements, e.g., “seven plants grew at least 7 cm.”

C The student made true statements about the range, “bumps,” “holes,” outlier, and median.

This work sample illustrates a standard-setting performance for the following parts of the standards:

M4 a Statistics and Probability Concepts: Collect and organize data to answer a question.
M4 b Statistics and Probability Concepts: Display data.
M4 c Statistics and Probability Concepts: Make statements and draw simple conclusions based on data.
M7 a Mathematical Communication: Use appropriate mathematical terms, vocabulary, and language.

• identifies and uses the mode necessary for making true statements, e.g., “more people chose red.”

D

• makes true statements based on a simple concept of average (median and mean), for a small sample size and where the situation is made evident with concrete materials or clear representations.

E The revision provides further evidence of the student's understanding.
Mathematical Communication: The student uses appropriate mathematical terms, vocabulary, and language, based on prior conceptual work.

Appropriate use of mathematical terms and vocabulary is evident in these parts of the work particularly, and is supported by evidence throughout the work.

**Height Measurement Statistics**

**Mathematics Communication**

The student uses appropriate mathematical terms, vocabulary, and language, based on prior conceptual work.

Appropriate use of mathematical terms and vocabulary is evident in these parts of the work particularly, and is supported by evidence throughout the work.

**Height Measurement Statistics**

**Line Plot**

- **Mathematics Communication**

  - The student uses appropriate mathematical terms, vocabulary, and language, based on prior conceptual work.
  
  Appropriate use of mathematical terms and vocabulary is evident in these parts of the work particularly, and is supported by evidence throughout the work.

  **Height Measurement Statistics**

  **Line Plot**
The task

After students had many chances to roll two dice and record the sums, the teacher gave the following instructions for this multi-part probability activity:

1. Think of a way to figure out all the combinations (sums) for two dice rolled together. Write a title and explain what you did. We will discuss the various strategies you came up with, and look at what works well.

2. Use a 6 x 6 grid to figure out the combinations for two dice sums to check that you found all the ways. (The teacher provided an example of a 6 x 6 grid, without supplying the numbers in the inner cells.) When you are finished, show how each sum can be shown as a fraction of the total number of combinations that are possible.

3. Play this game with a partner several times:
   - Draw a chart with spaces under the sums for two dice.
   - Draw 11 circles in any of the 11 spaces in any combination you want.
   - Write the reason why you put the circles where you did. Try to use fractions to explain your choices.
   - With a partner, take turns rolling the dice. Each time you roll a sum, put Xs in all the circles you have for that sum. You win when all your circles are crossed off.


Circumstances of performance

This sample of student work was produced under the following conditions:
- alone
- in a group
- in class
- as homework
- with teacher feedback
- with peer feedback
- timed
- opportunity for revision

Students had prior experience of making their own charts and using conventional fractional notation, and had explored probability using dice and other materials. The activities were spread out over two days.

What the work shows

- Arithmetic and Number Concepts: The student describes and compares quantities by using simple fractions...
  - All possible sums for paired dice are represented and compared as simple fractions.

- Statistics and Probability Concepts: The student predicts results, analyzes data, and finds out why some results are more likely, less likely, or equally likely...
  - The student organized the possible outcomes in order to make predictions about the likelihood of rolling particular two dice sums.

- Statistics and Probability Concepts: The student finds all possible combinations and arrangements within certain constraints involving a limited number of variables...
  - The student created a way to find all possible combinations for the sums of two dice.
Two Dice Sums

The top number is how many combinations there are out of the sum on top. The bottom number is how many combinations there are of two dice.

M6 Mathematical Skills and Tools: The student reads, creates, and represents data on...charts, tables....

A D The student-created charts, especially the horizontal tables under “Combinations” and the fractions chart, are appropriate, clear, and complete.

M7 Mathematical Communication: The student shows mathematical ideas in a variety of ways, including words, numbers, symbols,...charts,...tables....

D E The student used words to explain how the chart works. Numbers, symbols (“A” and “B”), and a table are used to communicate the combinations.

M7 c Mathematical Communication: The student explains solutions to problems clearly and logically, and supports solutions with evidence, in...written work.

B C The written explanations are sufficiently clear and logical for the elementary level. The student provided evidence in support of the game strategy by referring back to the data contained in the tables.

The work includes several errors in usage and grammar, e.g., “two dices,” “loosed” for “lost,” and “come” for “came.” This was a class assignment and was not intended to be further edited.
The task
The teacher asked the class to draw four Halloween characters (a ghost, a witch, a skeleton, and a pumpkin-headed scarecrow). She then instructed them to cut the figures into four heads and four bodies and to staple each set into a small “flip book.”

The teacher asked: How many characters could you possibly come up with by combining the different parts in various ways? Show and explain in detail all the combinations you could make.

Circumstances of performance
This sample of student work was produced under the following conditions:

- alone in a group
- in class as homework
- with teacher feedback with peer feedback
- timed opportunity for revision

The task was assigned as a two-day homework “problem of the week.” There had been no prior work by the class with similar discrete mathematics or logic tasks.

Mathematics required by the task
In this task the teacher instructed students to make and use a “flip book” as a tool to help find all possible combinations. So, while the task calls for extensive problem solving and reasoning, important decisions about what materials and approach to use in formulating the problem were made for the students.

For this reason the task does not provide an opportunity for students to demonstrate the formulation part (M5 a) of the Problem Solving and Reasoning standard. The task does, however, provide an opportunity for students to demonstrate the implementation (M5 b) and conclusion (M5 c) parts of the standard.

What the work shows
M4 f Statistics and Probability Concepts: The student finds all possible combinations and arrangements within certain constraints involving a limited number of variables.

A The student found all the possible combinations of the four heads and four bodies.
Problem Solving and Reasoning: The student makes the basic choices involved in planning and carrying out a solution; that is,...

- makes up and uses a variety of strategies and approaches to solving problems....

B The student used two approaches to solve the problem: a “flip book”/systematic listing and a student-created multiplicative formula.

- makes connections among concepts in order to solve problems....

C The student made connections among different conceptual approaches: “I know that my [systematic listing] solution is correct because there are four monsters, so that means there are four bodies [and four heads] and four times four equals sixteen.”

- solves problems in ways that make sense and explains why these ways make sense....

B C D The student explained why the solution made sense in several ways by setting out clearly both of the approaches used.
Creatures

M5 c Problem Solving and Reasoning: The student moves beyond the particular problem by making...extensions, and/or generalizations; for example,...makes the solution into a general rule that applies to other circumstances.

E The student moved beyond the particular problem by defining a general rule (four heads times four bodies) and applying it in a different circumstance: “I think if there were five cards there would be 25 ways.”

M7 c Mathematical Communication: The student shows mathematical ideas in a variety of ways, including words, numbers, symbols, pictures....

A C The systematic listing makes use of both pictures and sentences. Numbers and a diagram are used to represent the multiplicative formula.

M7 c Mathematical Communication: The student explains solutions to problems clearly and logically, and supports solutions with evidence....

A C D The explanation is clear, logical, and supported with diagrammatic, numeric, pictorial, and narrative evidence.

The work includes several misspelled words in the written portion of the work (e.g., “skeliton,” “pumkin,” and “carts” instead of “charts”). This was a homework assignment and was not intended to be further edited for spelling.
Work Sample & Commentary: ¿Se hace un triangulo?

The task
The teacher gave the students the following instructions:
Suppose you were given a string that is sixteen inches long. If you cut or fold it in any two places, will it always make a triangle?

Circumstances of performance
This sample of student work was produced under the following conditions:
- alone
- in group
- in class
- as homework
- with teacher feedback
- with peer feedback
- timed
- opportunity for revision

This sample was completed in Spanish in a bilingual classroom. The translation was provided by the teacher.

What the work shows
- **M2 a** Geometry and Measurement Concepts: The student uses many types of figures (angles, triangles...) and identifies the figures by their properties.
  - A B C The student used angles and triangles throughout this problem solving activity.
  - D The student did not merely identify, but discovered a property for, triangles: “if the two sides add up to more than the bottom side, it will make a triangle.”
- **M2 e** Geometry and Measurement Concepts: The student solves problems by showing relationships between and among figures.
  - A B C The student searched for and showed relationships among sides of triangles.
  - D The student summarized relationships among sides of triangles in a general rule.
¿Se hace un triángulo?

M2a Geometry and Measurement Concepts: The student uses basic ways of estimating and measuring the size of figures and objects in the real world, including length,…perimeter….
A B C

M2i Geometry and Measurement Concepts: The student selects and uses units, both formal and informal as appropriate, for estimating and measuring quantities such as…length….
B C

M5a Problem Solving and Reasoning: Formulation. Given the basic statement of a problem situation, the student:
• makes the important decisions about the approach, materials, and strategies to use, i.e., does not merely fill in a given chart, use a pre-specified manipulative, or go through a predetermined set of steps.
A The student decided to use a trial-and-error approach, which later became more systematic. The student also decided to use string, scissors, and a ruler.
• uses previously learned strategies, skills, knowledge, and concepts to make decisions.
A The student used knowledge about triangles and measurement to develop the approach.
• uses strategies, such as using manipulatives or drawing sketches, to model problems.
A B

M5b Problem Solving and Reasoning: Implementation. The student makes the basic choices involved in planning and carrying out a solution; that is, the student:
• makes up and uses a variety of strategies and approaches to solving problems and uses or learns approaches that other people use, as appropriate.
B The student used measurement strategies to solve the problem.
• makes connections among concepts in order to solve problems.
B C The student made connections between measurement and geometry concepts to solve the problem.
• solves problems in ways that make sense and explains why these ways make sense, e.g., defends the reasoning, explains the solution.
A B C

M5c Problem Solving and Reasoning: Conclusion. The student moves beyond a particular problem by making connections, extensions, and/or generalizations; for example, the student:
• makes the solution into a general rule that applies to other circumstances.
D The student went beyond the problem by finding a rule that would work for making any triangle.
¿Se hace un triangulo?

Translation

Page 1: I have a piece of string that measures 10 inches. If I fold it in two pieces, will it always form a triangle?

A → First I found that like this you can’t make a triangle.

I found that when I cut it in two parts and put in on the two corners you can’t make a triangle.

Later I discovered that if I put the short string near the long string, I could make a triangle.

To not be able to make a triangle I cut short sides. I did the same when I folded it.

B → Page 2: To not be able to make a triangle I must cut or fold 4 inches on each side. Then the largest side will be 6 inches. Then if you try to make a triangle you can’t. But if you cut the sides larger, like 6 inches each side, you will be able to make a triangle. I had an idea. If 4 inches plus 4 inches equals 8 inches, then I could put everything that equals 8. I tried it, it worked. I only show two ways that don’t make a triangle, but there are more. Everything that equals 8 will not make a triangle.

C → If you put 1, 1 1/2, 4, 1 1/2, 4 it will make a triangle. But if you put 6, 4 and 4 it will not make a triangle. So if I put here (arrow pointing to base of triangle) a number less than 6, then it will make a triangle. The rule is that if the two sides add up to more than the bottom side, it will make a triangle.

D →

Mathematical Skills and Tools: The student refers to geometric shapes and terms correctly with concrete objects or drawings, including triangle...side...

A B C D

Mathematical Communication: The student shows mathematical ideas in a variety of ways, including words, numbers, pictures, diagrams...

A B C D

Mathematical Communication: The student explains solutions to problems clearly and logically, and supports solutions with evidence, in both written and oral work.

A B C D
Work Sample & Commentary: The Great Fish Dilemma

The task
The teacher gave students the following written prompt:
How many different ways can you put nine fish in two bowls?
Show all your work and at the end explain why you made the decisions you did as you solved the problem.

Circumstances of performance
These samples of student work were produced under the following conditions:

- alone
- in class
- with teacher feedback
- timed
- in a group
- as homework
- with peer feedback
- opportunity for revision

Students could choose to include their work in the Vermont statewide portfolio assessment. They had been instructed in the specific criteria that the Vermont system uses in scoring problem solving work, so they knew what was expected. (See Marge Petit and Beth Hulbert, Learning How to Show Your Best, Exemplars, RR1, Box 7390, Underhill, VT 05489.)

What the work shows

MS a Problem Solving and Reasoning: Formulation. Given the basic statement of a problem situation, the student:
- makes the important decisions about the approach, materials, and strategies to use, i.e., does not merely fill in a given chart, use a pre-specified manipulative, or go through a predetermined set of steps.

A B C In Sample 1, the student made a chart. In Sample 2, the student used cubes and blocks, and then drew pictures.

- uses previously learned strategies, skills, knowledge, and concepts to make decisions.

A B In Sample 1, the student used an organized chart as a strategy. In Sample 2, the student used cubes and blocks to make decisions.

These work samples illustrate standard-setting performances for the following parts of the standards:

- MS a Problem Solving and Reasoning: Formulation.
- MS b Problem Solving and Reasoning: Implementation.
- MS c Problem Solving and Reasoning: Conclusion.

- uses strategies, such as using manipulatives or drawing sketches, to model problems.

ABC

MS b Problem Solving and Reasoning: Implementation. The student makes the basic choices involved in planning and carrying out a solution; that is, the student:
- makes up and uses a variety of strategies and approaches to solving problems and uses or learns approaches that other people use, as appropriate.

A B C D E In Sample 1, the student analyzed the chart to refine the solution. In Sample 2, the student used cubes, blocks, pictures, patterns, and a concept of “dividing” to solve the problem.

- makes connections among concepts in order to solve problems.

A C D Both students made connections between number concepts and the probability concept of finding all possible combinations. In Sample 2, the student also connected the concept of patterns to the concept of finding all possible combinations.

- solves problems in ways that make sense and explains why these ways make sense, e.g., defends the reasoning, explains the solution.

E F In Sample 1, the student explained why all of the possibilities “aren’t good.” In Sample 2, the student explained why the problem really only involved “dividing 7 fish.”
The Great Fish Dilemma

Problem Solving and Reasoning: Conclusion.
The student moves beyond a particular problem by making connections, extensions, and/or generalizations; for example, the student:

- explains a pattern that can be used in similar situations.
- explains how the problem solution can be applied to other school subjects and in real world situations.

In Sample 1, the student related the solution to the real world situation of cleaning the fishbowl. In Sample 2, the student related the solution to the real world situation of overcrowding the fish and what would happen when the fish had “babies.”

Each sample includes some misspelled words, such as “choises” for “choices” and “read” for “red.” The work was completed as a class exercise and was not edited to correct spelling mistakes.
The task

The teacher gave the students the following instructions:

Five people enter a room and introduce themselves to each other. If everyone shakes everyone else’s hand just once, what is the total number of handshakes that occurred?

For Sample 1, the instructions were given in written form. For Sample 2, the instructions were given orally.

Circumstances of performance

These samples of student work were produced under the following conditions:

- alone
- in class
- with teacher feedback
- timed
- in a group
- as homework
- with peer feedback
- opportunity for revision

This problem was a non-routine problem for the students in both of the classrooms, which made this an appropriate problem solving task.

Sample 2 was completed in Spanish in a bilingual classroom. The translation was provided by the teacher.

What the work shows

M3b Function and Algebra Concepts: The student builds iterations of simple non-linear patterns....

A B C D E F G In both samples, the students build the solution by building the pattern. They both extend this pattern into a general rule as well.

M4f Statistics and Probability Concepts: The student finds all possible combinations and arrangements within certain constraints involving a limited number of variables.

A B H I The drawings throughout both samples show that the students found all the possible combinations of handshakes.

M5a Problem Solving and Reasoning: Formulation. Given the basic statement of a problem situation, the student:

- makes the important decisions about the approach, materials, and strategies to use, i.e., does not merely fill in a given chart, use a pre-specified manipulative, or go through a predetermined set of steps.

A B H I Both students decided to use diagrams, lists, and equations.

- uses previously learned strategies, skills, knowledge, and concepts to make decisions.

A B H I Both samples demonstrate use of simple addition, as well as skill in drawing diagrams and making lists.

- uses strategies, such as using manipulatives or drawing sketches, to model problems.

A B H I
How Many Handshakes?

Five people enter a room and introduce themselves to each other. If everyone shakes everyone else’s hand just once, what is the total number of handshakes that occurred?

\[4 + 3 + 2 + 1 = 10\]

At first I made a chart using five bubbles numbered each. I did this to find the answer. Number one would shake 4 not shaking his own hand. I then drew four lines to 2, 3, 4 and 1. Then I did the same sort of thing on 4, 3 and 2. And 1. I did not shake my own hand and it equaled 10. That is how many hand shakes in all.

\[5 + 4 + 3 + 2 + 1 = 15\]

I noticed a pattern. Since you don’t have 6 shake any hands, you write 5, then add 4 then 3 then 2 then 1 and then it equals 15. Then 6 and then 5 and then it equals 15. That is what I did on the pattern. I thought where 7 people I would just have to make a math sentence like this:

\[6 + 5 + 4 + 3 + 2 + 1 = 21\]

A way you can always figure it out without a chart is by using a math sentence, starting at the highest of your number or the lowest, going up or down then adding all of them together. Remember not to include the person who can’t shake his own hand. For example if you had 29 people you might start with 29 because 29 cannot shake his own hand. Then you would count down to 1. Then add all the numbers together and that would equal the correct answer.

M5b Problem Solving and Reasoning: Implementation. The student makes the basic choices involved in planning and carrying out a solution; that is, the student:

- makes up and uses a variety of strategies and approaches to solving problems and uses or learns approaches that other people use, as appropriate.

A B H I Both students used diagrams, lists, and equations.

- makes connections among concepts in order to solve problems.

A B C E F H I Both students made connections between simple number concepts and patterns to solve the problem.

- solves problems in ways that make sense and explains why these ways make sense, e.g., defends the reasoning, explains the solution.

A B C E F H I J
How Many Handshakes?

MS c Problem Solving and Reasoning: Conclusion. The student moves beyond a particular problem by making connections, extensions, and/or generalizations; for example, the student:

- makes the solution into a general rule that applies to other circumstances.

DG Both students went beyond the problem by finding a rule that would work for any number of people.

M7 b Mathematical Communication: The student shows mathematical ideas in a variety of ways, including words, numbers,...pictures,...diagrams....
How Many Handshakes?

Sample 2 continued

7 personas saludan a uno al otro solo-mente una vez. ¿Cuántos saludos son?

\[ 6 + 5 + 4 + 3 + 2 + 1 + 0 = 21 \]

E

First I made 5 faces and then I put how many handshakes. After that I put that number 1 shakes the hand of 2, 3, 4, and 5. And number 2 shakes 3, 4, and 5 and number 3 shakes 4 and 5 and number 4 shakes 5 and number 5 shakes nobody's hand. I added the handshakes and the answer is 10.

\[ 4 + 3 + 2 + 1 + 0 = 10 \]

F

First I put 7 faces and then I put how many handshakes. I put that number 1 shakes the hand of 2, 3, 4, 5, 6 and 7 and also I did the same with 2, 3, 4, 5, 6 and 7. After that I added:

\[ 6 + 5 + 4 + 3 + 2 + 1 + 0 = 21 \]

G

It is easy. I only have to start with the number that is one less and add the rest. For example: If there are 39 I have to start with 38.

Sample 2 Translation

M7 e Mathematical Communication: The student explains solutions to problems clearly and logically, and supports solutions with evidence, in both written and oral work.
Work Sample & Commentary: Tangram Dispute

The task
The teacher gave students the prompt (the diagrams were the size of tangram pieces) illustrated here.

Circumstances of performance
This sample of student work was produced under the following conditions:

- alone
- in class
- with teacher feedback
- timed

- in a group
- as homework
- with peer feedback
- opportunity for revision

Students had previously worked with tangrams. They had begun to study area relationships and fractions.

This student was preparing a New Standards Elementary Mathematics Portfolio and was aware of the requirements for the portfolio. In providing evidence of problem solving work, students using New Standards mathematics portfolios must show that they can go beyond the problem and make connections, extensions or generalizations (MS 4).

What the work shows

\[ M1 \] Arithmetic and Number Concepts: The student describes and compares quantities by using concrete and real world models of simple fractions; that is:

- finds simple parts of wholes.

\[ A \] The student explained and showed in a model that there can be four equal (parallelogram) parts within the whole (parallelogram).

- uses drawings, diagrams, or models to show what the numerator and denominator mean, including when adding like fractions, e.g., \( \frac{1}{2} + \frac{1}{4} \), or when showing that \( \frac{3}{4} \) is more than \( \frac{1}{2} \).

\[ B \] The student used two models to show that one small parallelogram is equivalent to one out of four equal parts of the whole parallelogram, i.e., that “Tracy’s shape was \( \frac{3}{4} \) not a \( \frac{1}{2} \) of Terri’s shape.”
Tangram Dispute

M2  Geometry and Measurement Concepts: The student solves problems by showing relationships between and among figures, e.g., using congruence and similarity.

D  The student explained and modeled that “…one big shape can equal (i.e., be congruent to) four little of the same (i.e., a similar) shape.”

M5  Problem Solving and Reasoning: Formulation. Given the basic statement of a problem situation, the student:

• makes the important decisions about the approach, materials, and strategies to use, i.e., does not merely fill in a given chart, use a pre-specified manipulative, or go through a predetermined set of steps.

A  B  C  D  Given a problem situation with many possible approaches, the student decided what materials to use (pattern blocks and tiles) and what approach to follow.

• uses previously learned strategies, skills, knowledge, and concepts to make decisions.

A  The student used knowledge about the relationship between simple parts and wholes to decide who was correct in the dispute.

• uses strategies, such as using manipulatives or drawing sketches, to model problems.

B  C  D

M5  Problem Solving and Reasoning: Implementation. The student makes the basic choices involved in planning and carrying out a solution; that is, the student:

• makes up and uses a variety of strategies and approaches to solving problems and uses or learns approaches that other people use, as appropriate.

B  C  The student used fractional and geometric strategies to solve the problem.

• makes connections among concepts in order to solve problems.

A  B  C  D  The student made connections between number and geometry concepts to solve the problem.

• solves problems in ways that make sense and explains why these ways make sense, e.g., defends the reasoning, explains the solution.

A  B  C
Tangram Dispute

M5 e Problem Solving and Reasoning: Conclusion. The student moves beyond a particular problem by making connections, extensions, and/or generalizations; for example, the student:

• explains how the problem is similar to other problems he or she has solved....

D The student went beyond the problem by illustrating that the same approach and concepts can be used in a different situation.

M6 e Mathematical Skills and Tools: The student refers to geometric shapes and terms correctly with concrete objects or drawings, including triangle,... parallelogram....

B The student referred correctly to the parallelogram, which is not mentioned by name in the task.

M7 b Mathematical Communication: The student shows mathematical ideas in a variety of ways, including words, numbers,... pictures....

A B C D

M7 e Mathematical Communication: The student explains solutions to problems clearly and logically, and supports solutions with evidence, in both written and oral work.

A The solution is explained clearly.

B C The conclusion is supported by diagrammatic evidence.
Work Sample & Commentary: *Feverish Freddy*

**The task**
The teacher gave students the following prompt:

Freddy, a very precise real estate appraiser, was sent to appraise some lots on a local property. Freddy appraised lot A for $88,000, but then came down with the flu and was unable to determine the fair market value of the other lots. Help Freddy determine the values so he does not lose his job!

Using the value Freddy determined for lot A, figure the fair value of each of the other lots in relation to lot A. Write a report to Freddy informing him of your mathematical determinations. Show Freddy how you reached your conclusion, so he will have faith in your determination. Represent your determination mathematically, and use appropriate math language and representations to support your conclusion. Share with Freddy any mathematically relevant observations you make, as well as any recommendations.

In the actual task, this diagram matched the size of pattern blocks.

**Circumstances of performance**
This sample of student work was produced under the following conditions:

alone \hspace{1cm} in a group

in class \hspace{1cm} as homework

with teacher feedback \hspace{1cm} with peer feedback

timed \hspace{1cm} opportunity for revision

Students had prior experience working with pattern blocks and were familiar with their fractional equivalents (with the hexagon equal to one whole).

**What the work shows**

- Arithmetic and Number Concepts: The student adds, subtracts, multiplies, and divides whole numbers, with and without calculators; that is:
  - analyzes problem situations and contexts in order to figure out when to add, subtract, multiply, or divide.

A The student, given a problem situation with many possible approaches, figured out which arithmetic operations to use in order to come to a correct solution (evident throughout the work, without citing a specific operation).

B solves arithmetic problems by relating addition, subtraction, multiplication, and division to one another.

### This work sample illustrates a standard-setting performance for the following parts of the standards:

- *M1 a* Arithmetic and Number Concepts: Add, subtract, multiply, and divide whole numbers.
- *M1 d* Arithmetic and Number Concepts: Describe and compare quantities by using simple fractions.
- *M2 e* Geometry and Measurement Concepts: Solve problems by showing relationships between and among figures.
- *M3 d* Function and Algebra Concepts: Use letters, boxes, or other symbols to stand for any number, measured quantity, or object in simple situations.
- *M5 a* Problem Solving and Reasoning: Formulation.
- *M5 c* Problem Solving and Reasoning: Conclusion.
- *M6 e* Mathematical Skills and Tools: Refer to geometric shapes and terms correctly.
- *M6 g* Mathematical Skills and Tools: Read, create, and represent data.
- *M7 b* Mathematical Communication: Show mathematical ideas in a variety of ways.

The quotations from the Mathematics performance descriptions in this commentary are excerpted. The complete performance descriptions are shown on pages 60–65.
**Feverish Freddy**

**M1** Arithmetic and Number Concepts: The student describes and compares quantities by using concrete and real world models of simple fractions; that is:
- finds simple parts of wholes....

**A**

**M2** Geometry and Measurement Concepts: The student solves problems by showing relationships between and among figures, e.g., using congruence....
- The student compared congruent fractional parts of hexagons to determine the relative value of each part.

**C**

**M3** Function and Algebra Concepts: The student uses letters, boxes, or other symbols to stand for any number, measured quantity, or object in simple situations with concrete materials, i.e., demonstrates understanding and use of a beginning concept of a variable.

**B**

**M5** Problem Solving and Reasoning: Formulation. Given the basic statement of a problem situation, the student:
- makes the important decisions about the approach, materials, and strategies to use, i.e., does not merely fill in a given chart, use a pre-specified manipulative, or go through a predetermined set of steps.

**A** The student decided which approach (finding simple parts of hexagons and determining their relative value) and material (pattern blocks) to use.
- uses previously learned strategies, skills, knowledge, and concepts to make decisions.

**A C** The student used knowledge of the fractional equivalents among pattern block pieces to make decisions about the value of each piece (i.e., each lot).
- uses strategies, such as using manipulatives or drawing sketches, to model problems.

**A**

**M5** Problem Solving and Reasoning: Implementation. The student makes the basic choices involved in planning and carrying out a solution; that is, the student:
- makes up and uses a variety of strategies and approaches to solving problems....

**A B** The student used pictorial, geometric, narrative, and arithmetic approaches in the solution.
- makes connections among concepts in order to solve problems.

**A C D** The student connected the fractional concepts with geometric concepts of congruence and whole number concepts of value to solve the problem.
- solves problems in ways that make sense and explains why these ways make sense, e.g., defends the reasoning, explains the solution.

**A** The student explained the process used to reach the solution, without stating explicitly how the exact dollar value of each lot was derived.
- The work provides evidence of making sense of what to do with the cents that were left over.
M5 Problem Solving and Reasoning: Conclusion. The student moves beyond a particular problem by making connections, extensions, and/or generalizations.

B The student extended the problem by listing algebraic equations that show the fractional relationships among the lots. Expressing mathematical relationships algebraically goes beyond what most elementary students know and can be expected to do.

M6 Mathematical Skills and Tools: The student refers to geometric shapes and terms correctly with concrete objects or drawings, including triangle, parallelogram.

A D

M6 Mathematical Skills and Tools: The student reads, creates, and represents data on...charts, tables, diagrams.

A D The student created a chart and diagram to organize and represent information.

M6 Mathematical Skills and Tools: The student uses...manipulatives, calculators...as appropriate, to achieve solutions.

A D The student used pattern blocks and a calculator (in “Cost” and “Total cost”) to achieve a solution. The teacher verified that the student used a calculator.

M7 Mathematical Communication: The student shows mathematical ideas in a variety of ways, including words, numbers, symbols, pictures, charts, graphs, tables, diagrams, and models.

A B C D
Work Sample & Commentary: Counting on Frank

The task
Students were asked to read Counting on Frank by Rod Clement and to write a letter to the author commenting on at least one example of the mathematical claims made.

Circumstances of performance
This sample of student work was produced under the following conditions:

- alone
- in class
- with teacher feedback
- timed
- in a group
- as homework
- with peer feedback
- opportunity for revision

Four students discussed the mathematics in Counting on Frank in depth before they wrote their analyses. The students spent class time as well as many recesses figuring out how to test the claims in the book. For example, the students discussed how to use the classroom sink to test the claim in the book about the bathroom filling up with water. The students worked together to develop strategies for testing the claims about the peas and the ball-point pen. The teacher encouraged the students’ discussions, and provided time and materials in school for them to work out their reasoning. The students met outside of class to work together on the writing. Students completed the writing individually, at home.

The students completed this activity near the end of the school year. Earlier in the year, the students had studied area and perimeter concepts in depth, as well as applications of multiplication and division in problem solving activities.

This work sample represents a standard-setting performance for the following parts of the standards:

- **M1 a** Arithmetic and Number Concepts: Add, subtract, multiply, and divide whole numbers.
- **M1 b** Arithmetic and Number Concepts: Demonstrate understanding of the base ten place value system and use this knowledge to solve arithmetic tasks.
- **M1 c** Arithmetic and Number Concepts: Estimate, approximate, round off, use landmark numbers, or use exact numbers, as appropriate, in calculations.
- **M1 d** Arithmetic and Number Concepts: Describe and compare quantities by using concrete and real world models of simple fractions.
- **M1 e** Arithmetic and Number Concepts: Describe and compare quantities by using simple decimals.
- **M1 f** Arithmetic and Number Concepts: Describe and compare quantities by using whole numbers up to 10,000.
- **M2 g** Geometry and Measurement Concepts: Use basic ways of estimating and measuring the size of figures and objects in the real world.
- **M2 i** Geometry and Measurement Concepts: Select and use units for estimating and measuring quantities.
- **M2 j** Geometry and Measurement Concepts: Carry out simple unit conversions.
- **M3 a** Problem Solving and Reasoning: Formulation.
- **M3 b** Problem Solving and Reasoning: Implementation.
- **M5 a** Problem Solving and Reasoning: Conclusion.
- **M6 a** Mathematical Skills and Tools: Add, subtract, multiply, and divide whole numbers correctly.
- **M6 b** Mathematical Skills and Tools: Estimate numerically and spatially.
- **M6 c** Mathematical Skills and Tools: Measure accurately.
- **M6 d** Mathematical Skills and Tools: Compute time.
- **M6 f** Mathematical Skills and Tools: Use +, -, x, ÷, /,...and . (decimal point) correctly in number sentences and expressions.
- **M7 a** Mathematical Communication: Use appropriate mathematical terms, vocabulary, and language.
- **M7 b** Mathematical Communication: Show mathematical ideas in a variety of ways.
- **M7 c** Mathematical Communication: Explain solutions to problems clearly and logically.
- **M7 d** Mathematical Communication: Consider purpose and audience when communicating about mathematics.
- **M7 e** Mathematical Communication: Comprehend mathematics from reading assignments and from other sources.
Counting on Frank

What the work shows

**M1 a** Arithmetic and Number Concepts: The student adds, subtracts, multiplies, and divides whole numbers, with and without calculators; that is:
- multiplies, i.e.,... uses simple rates.

**M1 b**-**F** The consistent and effective use of multiplication throughout the student work shows a deep understanding at the elementary level.
- divides, i.e., puts things into groups, shares equally; calculates simple rates.

**G H I J**
- analyzes problem situations and contexts in order to figure out when to add, subtract, multiply, or divide....

**K L M** The student successfully figured out how and when to use arithmetic in these multistep problems. The analysis is particularly strong because the situations were general claims made by a character in a book, i.e., there was no hint about how to proceed to test the claims.
- solves arithmetic problems by relating addition, subtraction, multiplication, and division to one another.

**E** The student added to complete a multiplication problem. The teacher also verified that the student divided 700 by 4 and multiplied the answer by 3 with a calculator to figure out that "three quarters of 700 equals 525."
- computes answers mentally....

**D G N O** The student, as verified by the teacher, computed some of the arithmetic mentally, here and in other places in the work.

**M1 b** Arithmetic and Number Concepts: The student demonstrates understanding of the base ten place value system and uses this knowledge to solve arithmetic tasks; that is:... uses knowledge about ones, tens, hundreds, and thousands to figure out answers to multiplication and division tasks, e.g., 36 x 10, 18 x 100, 7 x 1,000, 4,000 ÷ 4.

**D E N**

**M1 c** Arithmetic and Number Concepts: The student estimates, approximates, rounds off, uses landmark numbers, or uses exact numbers, as appropriate, in calculations.

**J N O P Q R** The student used a well-developed sense of when it is appropriate to round off or estimate in these situations. Elsewhere, the student successfully used exact numbers in calculations.

**M1 d** Arithmetic and Number Concepts: The student describes and compares quantities by using concrete and real world models of simple fractions; that is, finds simple parts of wholes....

**E** The student used coffee cups and measured out 4 cups. The computation ("three quarters of 700 is 525") is advanced for this level.

**M1 e** Arithmetic and Number Concepts: The student describes and compares quantities by using simple decimals....

**J** The student took part of the decimal answer to a division problem (13.13) and used the nearest whole number (13) to make sense of the given situation, i.e., bags of peas.
Counting on Frank

**M1** Arithmetic and Number Concepts: The student describes and compares quantities by using whole numbers up to 10,000; that is, connects ideas of quantities to the real world....

**K1L1** The student called the author's bluff by actually connecting the ideas in the book to the real world.

**A1C1J1** In each problem, the student dealt successfully with quantities larger than 10,000 (10,240 seconds; 43,680 peas; 13,500 feet). The clarity and thoroughness of communication provides evidence of understanding of these quantities and represents a strong performance for this level.

**M2g** Geometry and Measurement Concepts: The student uses basic ways of estimating and measuring the size of figures and objects in the real world, including length, width, perimeter, and area.

**P1** The student went beyond the standards by measuring the volume of the bathroom in cubic feet. The teacher verified that the students had constructed the formula for area themselves earlier in the year through conceptual activities. The students' motivation to test the claim about the bathroom led them to a discussion of how to measure the size of the sink and the bathroom, and they constructed the idea of "layers" of length times width. The teacher then supplied them with the formula $L \times W \times H$.

**Q1** The student demonstrated understanding of area by listing some of the "millions of possibilities...that are close" for 264 square feet. The student also demonstrated understanding by applying the concept of area to the size of a bathroom floor, and noticing that an area of 264 square feet would be "huge."

**D1S1** The student measured the length of the pen and the length of the line and used these measurements to solve the problem.

**M2i** Geometry and Measurement Concepts: The student selects and uses units, both formal and informal as appropriate, for estimating and measuring quantities such as length, area, volume, and time.

**D1S1** The student used centimeters to measure the pen, and feet to measure the line.

**B1** The student used a coffee cup to measure volume.

**T1** The student used seconds to measure time.
Geometry and Measurement Concepts: The student carries out simple unit conversions, such as between cm and m, and between hours and minutes.

The student converted peas per day to peas per year, seconds to minutes, minutes to hours, and feet to yards.

Problem Solving and Reasoning: Formulation. Given the basic statement of a problem situation, the student:

- makes the important decisions about the approach, materials, and strategies to use, i.e., does not merely fill in a given chart, use a pre-specified manipulative, or go through a predetermined set of steps.

Problem Solving and Reasoning: Implementation. The student makes the basic choices involved in planning and carrying out a solution; that is, the student:

- makes up and uses a variety of strategies and approaches to solving problems....

Problem Solving and Reasoning: Conclusion. The student moves beyond a particular problem by making connections, extensions, and/or generalizations....

The assignment asked the student to focus on one claim made in the book. The student chose to extend the assignment by analyzing and writing about more than one claim. This is a reasonable extension for elementary school level.

Mathematical Skills and Tools: The student adds, subtracts, multiplies, and divides whole numbers correctly; that is:

- knows single digit addition, subtraction, multiplication, and division facts.

The student showed a command of basic facts, which was needed to carry out these and other calculations.

- adds...numbers with several digits.

- multiplies and divides numbers with one or two digits.

- multiplies and divides three digit numbers by one digit numbers.

Mathematical Skills and Tools: The student estimates numerically and spatially.

Mathematical Skills and Tools: The student measures length, area,...height,...and volume accurately in both the customary and metric systems.

The teacher verified the accuracy of the measurements.
Counting on Frank

M6a Mathematical Skills and Tools: The student computes time (in hours and minutes)....
M6b Mathematical Skills and Tools: The student uses recall, mental computations, pencil and paper, measuring devices, manipulatives, calculators... and advice from peers, as appropriate, to achieve solutions....
M6c Mathematical Communication: The student explains solutions to problems clearly and logically, and supports solutions with evidence, in both oral and written work.

K L M

M7a Mathematical Communication: The student uses appropriate mathematical terms, vocabulary, and language, based on prior conceptual work.
K L M The student used many mathematical terms appropriately, including “estimate,” “measure,” “average,” “multiply,” “divide,” “seconds,” “minutes,” “area,” “long,” “high,” “equal,” “cm,” “ft.”

P The use of the term “cubic feet” and the formula for volume go beyond what is required at this level.

U The student wrote “yds” after 13,500, but this mislabeling did not interfere with the correct mathematics in the work.

M7b Mathematical Communication: The student shows mathematical ideas in a variety of ways, including words, numbers, and symbols....
K L M

M7c Mathematical Communication: The student comprehends mathematics from reading assignments and from other sources.
K L M The student comprehends mathematics from reading assignments and from other sources.

M7d Mathematical Communication: The student focuses consistently on the purpose of disputing the claims in the book and wrote directly to the author.

K L M

The student misspelled a few words (e.g., “definitely”) and made some grammatical mistakes (e.g., capitalizing “book” in the middle of the sentence). The teacher did not require the students to fix the errors.
Work Sample & Commentary: School Uniforms Project

The task
The teacher asked students to come up with a question for a data study that they would then carry out. This student chose to survey the children of her school about their opinions on the following questions:
Would you like to have uniforms at this school?
What colors would you like the uniforms to be:
• white and red?
• white and magenta?
• white and blue?

Circumstances of performance
This sample of student work was produced under the following conditions:
alone in a group
in class as homework
with teacher feedback with peer feedback
timed opportunity for revision

The student developed and carried out this project with three classmates, but wrote an individual report. The class had not previously worked with collecting survey data.

In this classroom, students had been using New Standards Elementary Mathematics Portfolios. The teacher intended for students to be able to use this data survey to satisfy requirements for the project exhibit of the portfolio. According to the instructions for the project exhibit, students completing a data study should do the following things:
• Think of a situation where data can help you make a decision or recommendation. Describe the situation and the question you will ask.
• Decide on the groups and numbers of people you will survey.
• Predict the results of your study using specific fractions, percents, numbers, etc.
• Collect and organize your data in a chart. You may need to reorganize it.
• Compare your result to your prediction. Write down what was the same or different.
• Organize your data into a chart, graph, or diagram that shows your result.
• Record how your results help you make your recommendation or decision.
• Write a report that includes your decision or recommendation. Include all your work.

This sample was completed in Spanish in a bilingual classroom. The translation was provided by the teacher.

This work sample illustrates a standard-setting performance for the following parts of the standards:

M1 a Arithmetic and Number Concepts: Add, subtract, multiply, and divide whole numbers.
M1 c Arithmetic and Number Concepts: Estimate, approximate, round off, use landmark numbers, or use exact numbers in calculations.
M1 d Arithmetic and Number Concepts: Compare quantities by using concrete and real world models of simple fractions.
M1 e Arithmetic and Number Concepts: Describe and compare quantities by using simple decimals.
M4 a Statistics and Probability Concepts: Collect and organize data to answer a question.
M4 b Statistics and Probability Concepts: Display data.
M4 c Statistics and Probability Concepts: Make statements and draw simple conclusions based on data.
M4 d Statistics and Probability Concepts: Gather data about an entire group or by sampling group members to understand the concept of sample.
M5 a Problem Solving and Reasoning: Formulation.
M6 a Mathematical Skills and Tools: Add, subtract, multiply, and divide whole numbers correctly.
M6 g Mathematical Skills and Tools: Read, create, and represent data.
M7 d Mathematical Communication: Consider purpose and audience when communicating about mathematics.
M8 a Putting Mathematics to Work: Data study.
School Uniforms Project

What the work shows

Arithmetic and Number Concepts: The student...divides whole numbers, with and without calculators; that is:
• divides, i.e., puts things into groups, shares equally; calculates simple rates.
A
• analyzes problem situations and contexts in order to figure out when to...divide.
A  The student figured out that it was necessary to divide the number of children in the sample by the number of grades in the school in order to get an equal sampling from each grade.

Arithmetic and Number Concepts: The student estimates, approximates, rounds off,...as appropriate, in calculations.
B  The student made the appropriate choice of rounding off in order to more easily compute 10%.

Arithmetic and Number Concepts: The student describes and compares quantities by using concrete and real world models of simple fractions; that is:
• uses beginning proportional reasoning and simple ratios....
C  The student used fractions to represent a simple concept of proportion (“%...96 out of 100 children said...”).

Arithmetic and Number Concepts: The student describes and compares quantities by using simple decimals; that is:
• recognizes relationships among simple fractions, decimals, and percents, i.e., that ½ is the same as 0.5, and ½ is the same as 50%, with concrete materials, diagrams, and in real world situations....
B

Arithmetic and Number Concepts: The student estimates, approximates, rounds off,...as appropriate, in calculations.
B  The student made the appropriate choice of rounding off in order to more easily compute 10%.

Arithmetic and Number Concepts: The student describes and compares quantities by using concrete and real world models of simple fractions; that is:
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• recognizes relationships among simple fractions, decimals, and percents, i.e., that ½ is the same as 0.5, and ½ is the same as 50%, with concrete materials, diagrams, and in real world situations....
B

Arithmetic and Number Concepts: The student estimates, approximates, rounds off,...as appropriate, in calculations.
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Arithmetic and Number Concepts: The student describes and compares quantities by using simple decimals; that is:
• recognizes relationships among simple fractions, decimals, and percents, i.e., that ½ is the same as 0.5, and ½ is the same as 50%, with concrete materials, diagrams, and in real world situations....
B
School Uniforms Project

M4 Statistics and Probability Concepts: The student collects and organizes data to answer a question or test a hypothesis by comparing sets of data.

D E The student collected data to test a hypothesis.

F G The student organized the data.

M4 Statistics and Probability Concepts: The student displays data in... graphs, tables, and charts.

F G

M4 Statistics and Probability Concepts: The student makes statements and draws simple conclusions based on data; that is:....

• compares data in order to make true statements....

C

• uses data, including statements about the data, to make a simple concluding statement about a situation....

H The student used the data to make simple concluding statements. Most of the statements made (e.g., “Most of the children...” for 96%) are accurate. One conclusion (“...almost all the girls...” for 25 out of 57 sampled) is not accurate.
School Uniforms Project

Mathematics and Probability Concepts: The student gathers data about an entire group or by sampling group members to understand the concept of sample, i.e., that a large sample leads to more reliable information.

I The reasoning behind choosing to sample roughly 10% of the school is clearly explained.

Problem Solving and Reasoning: Formulation. Given the basic statement of a problem situation, the student.

- makes the important decisions about the approach, materials, and strategies to use, i.e., does not merely fill in a given chart, use a pre-specified manipulative, or go through a predetermined set of steps.

A B D E F G The student made decisions about the sampling method, and the charts and graphs to be used.

- uses previously learned strategies, skills, knowledge, and concepts to make decisions.

B The student used the concept of percent in making decisions.

Mathematical Skills and Tools: The student adds, subtracts, multiplies, and divides whole numbers correctly; that is:

- multiplies and divides three digit numbers by one digit numbers.

A

Mathematical Skills and Tools: The student reads, creates, and represents data on...charts, tables, diagrams, and bar graphs.

D E F G

Mathematical Communication: The student considers purpose and audience when communicating about mathematics.

J The report enables readers to understand the project.
### School Uniforms Project

<table>
<thead>
<tr>
<th>E</th>
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**M3** - Putting Mathematics to Work: Data study, in which the student:
- develops a question and a hypothesis in a situation where data could help make a decision or recommendation.

**J K** - The student showed clearly that the question could lead directly to a real recommendation.

The project does not entirely fulfill this criterion for the data study, because the question is not expressed as a hypothesis. Nevertheless, the student accomplished the important purposes of a large scale mathematics project. The purpose of the Putting Mathematics to Work standard is for students to make important decisions about using mathematics and to put extensive mathematics to work in realistic situations.

- decides on a group or groups to be sampled and makes predictions of the results, with specific percents, fractions, or numbers.

**B I L** - The student used clear arithmetic and logical reasons in choosing the group and numbers used in the sample. The prediction applied to only one of the two questions posed. The prediction included no specific numbers. This lack of specificity does not of itself restrict the project from meeting the standard for a data study.

- collects, represents, and displays data in order to help make the decision or recommendation; compares the results with the predictions.

**F G H** - writes a report that includes recommendations supported by diagrams, charts, and graphs, and acknowledges assistance received from parents, peers, and teachers.
School Uniforms Project

Translation

Would you like to have uniforms at this school?

What colors would you like the uniforms to be?

white and red
white and yellow
white and blue

There, Ross, Corky, and I are going to ask the children in the school if they would like uniforms. If they answer yes, we are going to ask them what colors they would like the uniforms to be.

We think that is in important to know this because there are less problems and the children will respect one another.

The principal and the vice principal would like to know this because they would order them and because the children would wear them.

We have 586 children in the school rounded off to 1,000 and we are going to ask 10% or 100 children in the school in kindergarten through fifth.

At first we thought we were going to ask 10% but we changed our minds because it was hard for us to be asking questions of all the kids in the school and besides the children in kindergarten and first have a little trouble answering the questions. That's why we decided to ask only grades 2-5 and 25 children at each grade. But first we need to find out how many rooms there are at each grade, so we can ask the same quantity of children at each grade.

First we did some mathematics we divided 100 by 6 but there was a remainder of 4. Next we divided 100 by 5 and the result was 20 but the children in first grade have a hard time answering the questions. Finally we found the solution we divided 100 by 4 25 and we are going to idea the fraction for our survey.

I think that most of the boys are going to choose blue and white and the girls are going to choose red and white. But I would like it if they choose magenta and white.

Most of the children said that they would like to have uniforms only 4 children said that they would not like to have uniforms. Many boys and girls chose the colors blue and white, I guessed part of it had almost all the girls chose blue and white.

Many children chose that they would like uniforms 96 out of 100 children said they wanted uniforms. 96 out of 100 children did not want uniforms.

23 out of 96 children chose the color red and white
57 out of 96 children chose the color blue and white
57 out of 96 children chose the color magenta or magenta white.

I recommend to the principal and the vice principal that they order blue and white uniforms.

I put 56 for those because languages 4 children said that they didn't want uniforms and I subtracted 100 4 and the answer was 96. With the children that said they did not want uniforms we did not ask them what colors they would like.

Graph titled: Total of children who give opinions about uniforms

Graph titled: Opinions about the colors of the uniforms

Tally sheet for the surveying of the children by grade

Tally sheet for the surveying of the children separated by boy/girl
Work Sample & Commentary: Catapult Investigation

The task
After each student had built an individual catapult according to the teacher’s design, the teacher gave the following instructions:

Use your catapults to determine which setting (on the catapult) is best for shooting a wet sponge the furthest. To complete the project you will need to:

• design and carry out a test that determines the optimum setting for shooting a wet sponge the furthest;
• include all information that would be necessary for anyone reading this response to follow your work completely.

Circumstances of performance
This sample of student work was produced under the following conditions:

alone  in a group
in class    as homework
with teacher feedback    with peer feedback
timed      opportunity for revision

Students completed this project as part of a unit on simple machines and had explored levers and fulcrums. The class took several days to complete the testing of catapult-setting variables and to write up the results as a report. Students worked with partners in the testing of their catapults.

What the work shows

M4 a Statistics and Probability Concepts: The student collects and organizes data to answer a question...by comparing sets of data.

A B The student collected, organized, and compared data in order to determine which catapult setting sends wet sponges the furthest.

M4 b Statistics and Probability Concepts: The student displays data in line plots, graphs, tables, and charts.

A Data are displayed in a clear table.
B Data are displayed in a graph. The data for settings A1 and A2 were placed off line but this does not detract from the overall clarity of the graph.

M4 c Statistics and Probability Concepts: The student makes statements and draws simple conclusions based on data; that is...

• compares data in order to make true statements....

C The student compared the data to make true statements: “A2 was the best setting because (it) shot very far...It had an average of 123 inches...C2 is the worst setting....”

This work sample illustrates a standard-setting performance for the following parts of the standards:

M4 a Statistics and Probability Concepts: Collect and organize data to answer a question.
M4 b Statistics and Probability Concepts: Display data.
M4 c Statistics and Probability Concepts: Make statements and draw simple conclusions based on data.
M5 a Problem Solving and Reasoning: Formulation.
M5 b Problem Solving and Reasoning: Implementation.
M6 a Mathematical Skills and Tools: Measure accurately in both the customary and metric systems.
M6 g Mathematical Skills and Tools: Read, create, and represent data.
M6 h Mathematical Skills and Tools: Use measuring devices and calculators to achieve solutions.
M7 b Mathematical Communication: Show mathematical ideas in a variety of ways.
M8 b Putting Mathematics to Work: Science study.
Catapult Investigation

- makes true statements based on a simple concept of average (median and mean), for a small sample size.

A C

- uses data, including statements about the data, to make a simple concluding statement about a situation.

D

M4 Statistics and Probability Concepts: The student finds all possible combinations and arrangements within certain constraints involving a limited number of variables.

E In the course of testing the settings, the student found all the possible combinations of settings.

M5 Problem Solving and Reasoning: Formulation. Given the basic statement of a problem situation, the student:

- makes the important decisions about the approach, materials, and strategies to use, i.e., does not merely fill in a given chart, use a pre-specified manipulative, or go through a pre-determined set of steps.

A B C E F The student decided on the methods for determining, recording, and representing the most effective setting.
Catapult Investigation

M5 b Problem Solving and Reasoning: Implementation. The student makes the basic choices involved in planning and carrying out a solution; that is, the student:
• makes up and uses a variety of strategies and approaches to solving problems and uses or learns approaches that other people use, as appropriate.
A B C E F The student created an approach for testing the settings.
• makes connections among concepts in order to solve problems.
A E The student made connections among statistics concepts (e.g., finding all combinations of settings and averaging like results) to answer the question.
• solves problems in ways that make sense and explains why these ways make sense, e.g., defends the reasoning, explains the solution.
C D E F

M5 e Mathematical Skills and Tools: The student measures length...accurately in both the customary and metric systems.
A The student measured the lengths of firings. The teacher verified the accuracy of measurements.
Catapult Investigation

M6 b Mathematical Skills and Tools: The student uses...measuring devices...calculators,...as appropriate, to achieve solutions.
A The student used measuring devices to achieve a solution.
A The student used a calculator (as verified by the teacher) to compute averages.

M7 b Mathematical Communication: The student shows mathematical ideas in a variety of ways, including words, numbers, symbols, pictures, charts, graphs, tables, diagrams, and models.
A B F G H I The student showed his ideas in multiple ways. Some of the diagrams in particular are unlabeled, labeled confusingly, or difficult to interpret (see the three diagrams of the catapult). Overall, however, the ideas are communicated with sufficient clarity.
Putting Mathematics to Work: The student conducts a science study, in which the student:

- decides on a specific science question to study and identifies the mathematics that will be used, e.g., measurement.

The student investigated a science question and decided what mathematics should be used.

- develops a prediction (a hypothesis) and develops procedures to test the hypothesis.

This investigation does not entirely fit the criteria for a “Science Study,” because it does not ask the student for a hypothesis. The purpose of the Putting Mathematics to Work standard is for students to put extensive mathematics to work in realistic situations. The investigation clearly accomplishes this central purpose of a large scale mathematics project.

- collects and records data, represents and displays data, and compares results with predictions.

The data are recorded, represented, and compared. Since the teacher called for students to answer a question in this investigation, the results of the data collection are used to this end, rather than to make a prediction.

- writes a report that compares the results with the hypothesis; supports the results with diagrams, charts, and graphs; acknowledges assistance received from parents, peers, and teachers.

The report is clear and uses the results to answer the question. The teacher did not ask students to acknowledge assistance; nevertheless, the student acknowledged his partner’s assistance in making certain decisions.
The task

The teacher asked students to complete a large scale mathematics project chosen from among five kinds. This student chose to do a “Pure Mathematics Investigation.” The teacher gave the student the written instructions illustrated here.

Circumstances of performance

This sample of student work was produced under the following conditions:

- alone
- in class
- with teacher feedback
- timed

In this classroom students were using the New Standards Elementary Mathematics Portfolio. The teacher intended students to be able to use their chosen projects to fulfill the requirements for the project exhibit in the portfolio.

What the work shows

M3 b Function and Algebra Concepts: The student builds iterations of simple non-linear patterns, including multiplicative and squaring patterns (e.g., “growing” patterns) with concrete materials, and recognizes that these patterns are not linear.

A The student built (again and again) the non-linear pattern generated by this “trick” question (i.e., that the numbers appearing before a four turns up are always five or nine).

M4 a Statistics and Probability Concepts: The student collects and organizes data to answer a question...by comparing sets of data.

B The student represented data in a table in order to further explore the number relationships.

M5 a Problem Solving and Reasoning: Formulation. Given the basic statement of a problem situation, the student:

• makes the important decisions about the approach, materials, and strategies to use, i.e., does not merely fill in a given chart, use a pre-specified manipulative, or go through a predetermined set of steps.

A B C D E The student made all decisions regarding how to proceed with this entirely unformulated project.

M6 a Mathematical Skills and Tools: The student reads, creates, and represents data on charts, tables, diagrams....

B

M7 b Mathematical Communication: The student shows mathematical ideas in a variety of ways.

M7 c Mathematical Communication: Explain solutions to problems clearly and logically.

M7 d Mathematical Communication: Consider purpose and audience when communicating about mathematics.

M8 e Putting Mathematics to Work: Pure mathematics investigation.
The Never Ending Four

B C The student used a narrative description and tables to illustrate the nature of the “problem” (or, in this case, “trick”).

M7 Mathematical Communication: The student considers purpose and audience when communicating about mathematics.

D E The student clearly explained the nature of the project as well as how it would be conveyed to the classroom audience.

M8 Putting Mathematics to Work: The student conducts a pure mathematics investigation, in which the student:

• decides on the area of mathematics to investigate, e.g., numbers, shapes, patterns.

• describes a question or concept to investigate.

• decides on representations that will be used, e.g., numbers, symbols, diagrams, shapes, or physical models.

A B C D E

• carries out the investigation.

A B C D E

• writes a report that includes any generalizations drawn from the investigation, and acknowledges assistance received from parents, peers, and teachers.

A B C D E

Three errors in counting the letters (32, 13, and 30) do not detract from the project.
Work Sample & Commentary: *Dream House Project*

**The task**

The teacher gave students the following instructions, and clarified them through an extended discussion:

Plan and design your own “dream house” within certain cost constraints. Here are the budget, cost, and building rules you will have to work with:

- You will have $200,000 to spend altogether.
- The land for the house costs $100,000.
- Construction of “regular” rooms (traditional rooms with no special requirements) costs $75 per square foot.
- Construction of “special” rooms (requiring special wiring, plumbing, or unusual materials) costs $150 per square foot.
- All houses must include a kitchen, bathroom, bedroom, and living room.
- Rooms and hallways must have reasonable areas.
- The overall design must be convenient and practical (e.g., provide easy access to rooms, allow for privacy, include doors and hallways in practical locations, etc.).
- The number of sides to your floor plan should be limited, to avoid a sprawling, awkward design.

To get started, you will:

- Complete a rough draft of your floor plan by cutting out the rooms from half-inch graph paper.
- Record the calculations of the area and cost on the cut-out of the room.
- Track your budget on a running budget sheet (“Budget Update”).
- Meet in a response group to see if your rough draft is reasonable.

To finish, you will:

- Make a final draft of your floor plan without graph paper, using a ruler and protractor, and following the scale exactly.
- Draw a front and side view of your dream house based on your floor plan.

**Circumstances of performance**

This sample of student work was produced under the following conditions:

alone in a group
in class as homework
with teacher feedback with peer feedback
timed opportunity for revision

Students completed this project over the course of four weeks. They worked individually except when initially measuring the classroom as part of a pre-project activity (“Scale Model of Room 14”). They also met once in small peer response groups, and once with an editing partner.

The teacher reviewed students’ progress on this project daily, and created the “House Project Response Group Meeting Notes” and the “Editing Checklist for House Description” in order to help students complete the project.

The students had worked with two digit multiplication and the concept of area prior to beginning the project. They were allowed to use calculators to check calculations and for multiplication with multipliers of three or more digits.
What the work shows

M1 a Arithmetic and Number Concepts: The student adds, subtracts, multiplies...whole numbers with and without calculators; that is:

• adds, i.e., joins things together, increases.

A The student added costs of rooms to determine the amount remaining in the budget.

• subtracts, i.e., takes away, compares, finds the difference.

B The student subtracted to determine the amount of the budget remaining.

• multiplies, i.e., uses repeated addition, counts by multiples....

C The student multiplied the area of rooms by cost per square foot to determine the total cost of each room.

M2 b Geometry and Measurement Concepts: The student visualizes and represents two dimensional views of simple rectangular three dimensional shapes....

D The student visualized and represented the two dimensional house floor plan in side and front views, i.e., as if in three dimensions.

M2 g Geometry and Measurement Concepts: The student uses basic ways of estimating and measuring the size of figures and objects in the real world, including length, width, perimeter, and area.

E The student measured the length and width of the classroom in order to determine area.
Dream House Project

M2 Geometry and Measurement Concepts: The student uses scales for rectangular scale drawings based on work with concrete models and graph paper.

E C The student used half-inch square graph paper to create a ½ inch: 1 foot scale model of a room.

M4 Statistics and Probability Concepts: The student makes statements and draws simple conclusions based on data; that is, reads data in line plots.

F The student interpreted line plot data in order to make decisions about reasonable areas of rooms in the floor plan.

M6 Mathematical Skills and Tools: The student adds, subtracts, multiplies whole numbers correctly.

A B C There is evidence of accurate calculations in these parts of the work and throughout.
**Dream House Project**

**M6 e Mathematical Skills and Tools:** The student measures length, area, perimeter accurately in the customary system.

**C E** These parts of the work provide evidence of accurate calculations that are supported by the calculations shown in the rough and final drafts.

**M6 h Mathematical Skills and Tools:** The student uses measuring devices graded appropriately for given situations, such as rulers (customary to the 1/8 inch...), graph paper (customary to the inch or half-inch...). 

**C E G** The student used a ruler and graph paper to measure lengths of sides of rooms.

**M7 a Mathematical Communication:** The student uses appropriate mathematical terms, vocabulary, and language, based on prior conceptual work.

**H** The student used appropriate mathematical vocabulary to describe the process involved in completing the project, e.g., “area,” “unit,” “reasonable.”
Dream House Project

M7b Mathematical Communication: The student shows mathematical ideas in a variety of ways, including words, numbers...graphs...tables...

B E F H

M7d Mathematical Communication: The student considers purpose and audience when communicating about mathematics.

The student communicated the purpose and explained the process for completing the project clearly enough for the project to be understood by an otherwise uninformed peer.
Dream House Project

**Question 1:** Explain the project... What is the? What's about? How much did you spend? How much does one square foot cost? What rooms did you have to have? What's the scale (how long is one foot on your plan)?

The project is a house project. It’s about building a house and then drawing it. The total cost was $45,000, and the land cost $10,000. I drew the house and the land together including the house and the land. I used a square room like a bedroom or a bathroom, and the squares that make up a square inch is 1 square foot.

**Question 2:** Explain about area... What's area? What units do you use to measure area? Give a clear example - one of your rooms to explain how you know how to figure the area.

The area of how big is the room thing is the square feet. It is: 10 x 10 = 100 square feet. I did it by multiplying the length on one side and then I counted how many square feet are on the 100 then I multiply it together and I got the answer of the area.

**Question 3:** Explain: 1. How many square feet does your finished house have? 2. How much does it cost altogether (explain where your calculations are and so they can be checked)? 3. How much money is leftover in your budget? (explain how you figured the cost).

My finished house have 1082 square feet. Because I added up every room's square feet and I got the answer. Her I marked it with the other answer when I wrote down my exact square feet cost of my whole house and then I marked it with $75,800. Then I used the calculator and got the answer of 1082 square feet cost. I used the calculator to get the answer of 1082 square feet cost. I used the calculator to get the answer of $75,800 and I used the calculator to check it with the other answer. The one when I wrote down my exact square feet cost of my whole house and then I marked it with $75,800. Then I used the calculator to get the answer of 1082 square feet cost.
Dream House Project

There are several spelling and grammatical errors in the rough draft work. Only the final draft description and floor plan were edited for spelling and grammar. A few errors remain in the final draft work; however, the errors do not detract from its accuracy or clarity (e.g., “mutiply” for “multiply”).
Dream House Project

The project is a house project. It is about building our own dream house. The total we could spend for our house is $100,000 and the land cost $200,000. The land is not free, of course. So all together including the house and the land, it cost $200,000. One square foot cost $150.00 for a regular room like a bedroom or a living room, but one square foot cost $200.00 for a special room like a swimming pool. I just made it up in the design.

Here is space covered in the whole thing, the rule one square foot as one unit. My bedroom is 400 square feet, so I did it by multiplying it. I counted how many square feet there were on one side and then I counted how many feet there were on the top, then I multiply it together, and I got the answer. Now I want to know how many square feet there are in the whole room, I do that with all the other rooms too.

If one of my rooms is 12 ft x 14 ft. If first do 12 x 4 = 48. Then I write the 48 and add that to the 10 to the right of 48 so it will be 58. Then I do 3 x 10 equals 30, I write the 3 under the 5 and I write the 10 on the left side of 3. Then I add the 52, the answer of 182 square feet.

To find out the cost of a square foot, you multiply the total square feet by $200,000 for the total square feet. I do the total square feet of the is a special room, so if the room is 10 ft x 10 ft, the total square feet of that room times $150.00, then I add the how much one square foot cost got.

I know all my rooms have a reasonable size because it looked at the rooms chart that my class had made those rooms are real rooms that the class had measured. Anyway, I compared my rooms to the chart. The class had made and see if it is too big or too small or just perfect. My rooms always turn out to be perfect. Another way I could tell is by cutting out fake furniture and put in the rooms and see it fits.

My finished house have 1040 square feet because I added up all the rooms’ total square feet and I got the answer. Then I check it with the other paper where I wrote down my exact square feet of my whole house and it is the same. That is how much square feet I need to calculate, I get the answer. It was $82000. That is how much money I spend. I still have $22000 left.

Other sheet of paper. The whole perimeter of my house is 142 ft. I did it by measuring the walls of my house and adding it together.
Dream House Project
The task
The teacher gave students the following instructions, and clarified them through an extended discussion:

Look at the polyhedra on this poster. Choose one of them to construct. You will be able to use a diagram of the net in order to help you see how the shapes fit together. You will also:

• Choose a length for the side of the polygons that will be the faces of your polyhedron.
• Use a compass to make a template of the polygons that make the faces of your polyhedron. The main face you will use will be an equilateral triangle.
• Extend the sides to make flaps, and then join the faces together by using rubber bands on the flaps.
• Write about your construction.

The main choices of polyhedra to construct included the regular octahedron (a figure with eight equilateral triangles as faces), the regular icosahedron (a figure with 20 equilateral triangles as faces), and the cuboctahedron (a figure with six squares and eight triangles as faces).

The student in this sample constructed a snub icosadodecahedron. This polyhedron was not offered as a choice for students to construct, because it was considered too complex for elementary students. The snub icosadodecahedron is a polyhedron made of 80 triangles and 12 pentagons. (Its name is derived from the dodecahedron, which is composed of pentagons, and the icosahedron, which is composed of equilateral triangles.)

In order to complete this task, the student voluntarily:

• figured out the net for the polyhedron, with assistance from the teacher;
• received additional instruction from the teacher about how to use a protractor to measure the angles and how to construct a pentagon;
• spent several hours beyond the allotted time for the project, including before and after school and during lunch recesses.

Circumstances of performance
This sample of student work was produced under the following conditions:

alone in a group
in class as homework
with teacher feedback with peer feedback
timed opportunity for revision

Before beginning this project, students had been working with two dimensional geometry concepts, especially symmetry, congruence, quadrilaterals, triangles, circles, and diameter. They had constructed two dimensional “stained glass windows,” according to specifications about the geometry required.

This student’s work illustrates parts of the standard for Geometry and Measurement Concepts. In addition, the sample helps to illustrate what it means for an elementary student to exceed the standard in some areas, e.g., the student constructs and works with pentagons, measuring the angles of the pentagon in degrees.
Constructing a Polyhedron

What the work shows

M2 a Geometry and Measurement Concepts: The student uses many types of figures (angles, triangles,...polygons) and identifies the figures by their properties.
A The student used triangles; her use of the pentagon and snub icosadodecahedron represents work that exceeds the standard for this level.

M2 b Geometry and Measurement Concepts: The student extends and creates geometric patterns using concrete and pictorial models.
B The student found a pattern in this polyhedron.

M2 c Geometry and Measurement Concepts: The student uses basic ways of estimating and measuring the size of figures and objects in the real world.
A C In order to construct the triangles and pentagons, the student measured the length of the sides.

M2 d Geometry and Measurement Concepts: The student selects and uses units, both formal and informal as appropriate, for estimating and measuring quantities such as...length.
A The use of inches is appropriate for this level; the use of degrees exceeds the standard for this level.

M6 a Mathematical Skills and Tools: The student estimates numerically and spatially.
A D The successful completion of the icosadodecahedron provides evidence that the student estimated spatially and numerically, as both kinds of estimation were clearly required in the construction.

M6 b Mathematical Skills and Tools: The student measures length...accurately in...the customary...
A C

M6 c Mathematical Skills and Tools: The student measures length...accurately in...the customary...

M6 e Mathematical Skills and Tools: The student refers to geometric shapes and terms correctly with concrete objects or drawings, including triangle,...side,...face,...point, line...; and refers with assistance to...polygon, polyhedron, angle, vertex...
E The student refers correctly to angle, triangle, pentagon, and icosadodecahedron. The student appears to have a working understanding of the term “vertex” but has not yet developed a clear definition. (The writing of “4½” is also incorrect in some places.)

M6 f Mathematical Skills and Tools: The student uses...measuring devices...to achieve solutions.
A C The use of a ruler is appropriate for this level; the use of the compass and protractor exceeds the standard.
If this project were written up in a more systematic way, it would illustrate an example of a pure mathematics investigation, one of the projects described in M6.

Several words are misspelled in the written portion of the work. This was a class assignment and was not intended to be further edited for spelling or grammar. The focus of the assignment was the construction. The spelling and grammar errors are reasonable for first draft work, e.g., “notice” for “noticed” and “pentigon” for “pentagon.”

Readers who build this figure will discover that there are actually 80 triangles, not 78, as the student reports.
Constructing a Polyhedron

The side of my triangle is 4 cm because the side of the pentagon is 4 cm and the side of both pentagons had to be the same. So first I made a 4 cm line, then I took my compass and put the point on the end of the line I made and I made a 4 cm arc. Then I made the other arc and the end of the line was made a 4 cm arc on the other end. Then I made a line at the middle of the x 4 cm line in one end of the 4 cm line. And then I did the same to the other side. And that's how I made my triangle.